

---

# **Introduction to Perspective Geometry Brunelleschi's Experiment**

---

**Visual Imaging in the Electronic Age**

Donald P. Greenberg

September 8, 2020

Lecture #2

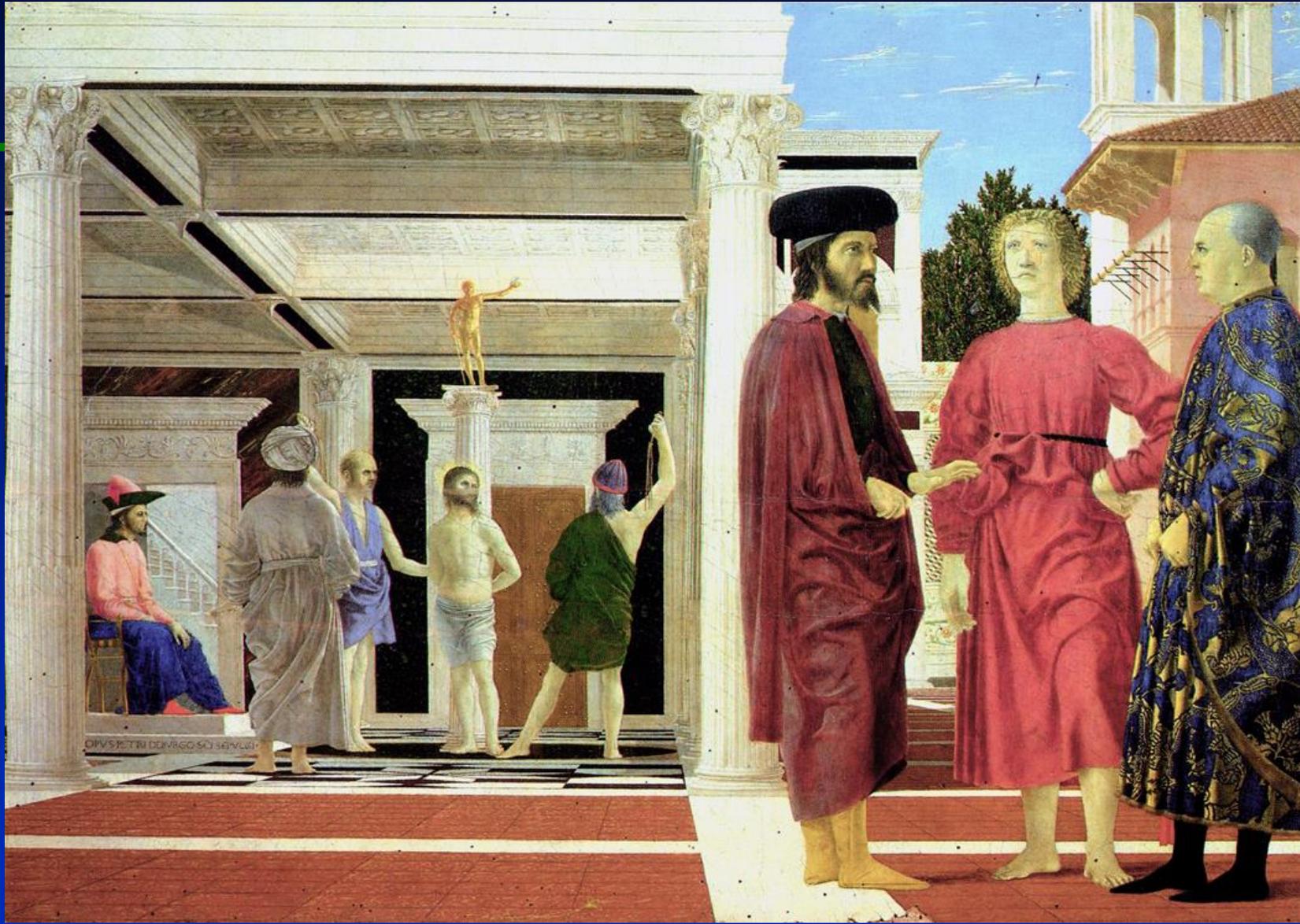
---

- Required Reading:

Art Perspective Handout: PDF

- References:

- Martin Kemp. "The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat" Yale University Press
- Ingrid Carlbom , Joseph Paciorek. "Planar Geometric Projections and Viewing Transformation," Computing Surveys, vol. 10, no. 4, December 1978



1458

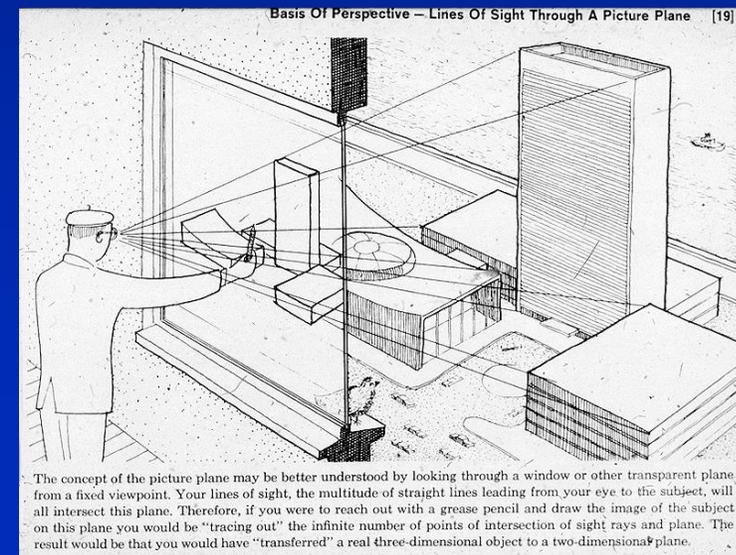
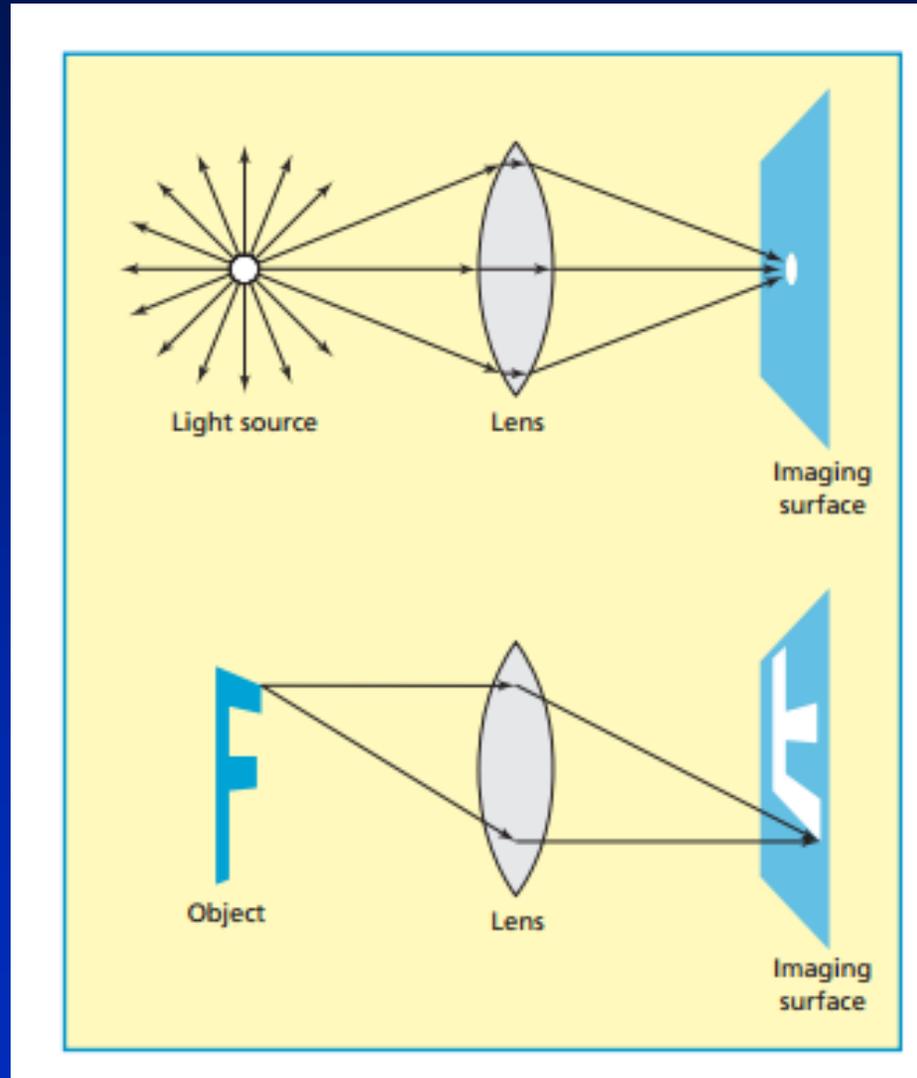
**The Flagellation of Christ**, c. 1458-60. Piero della Francesca. Tempera. 59 x 81.5 cm. Urbino, Galleria Nazionale delle Marche.



1877

Gustave Caillebotte, **Paris Street; Rainy Day** From *Charles H. and Mary F. S. Worcester Collection*, Oil on canvas

# Light as Rays





Albrecht Durer. **Untitled** (Artist using a glass to take a portrait). From *Underweysung der Messung mit dem Zirkel und Riichtscheyt*, 1st Ed, 1525. Woodcut print.

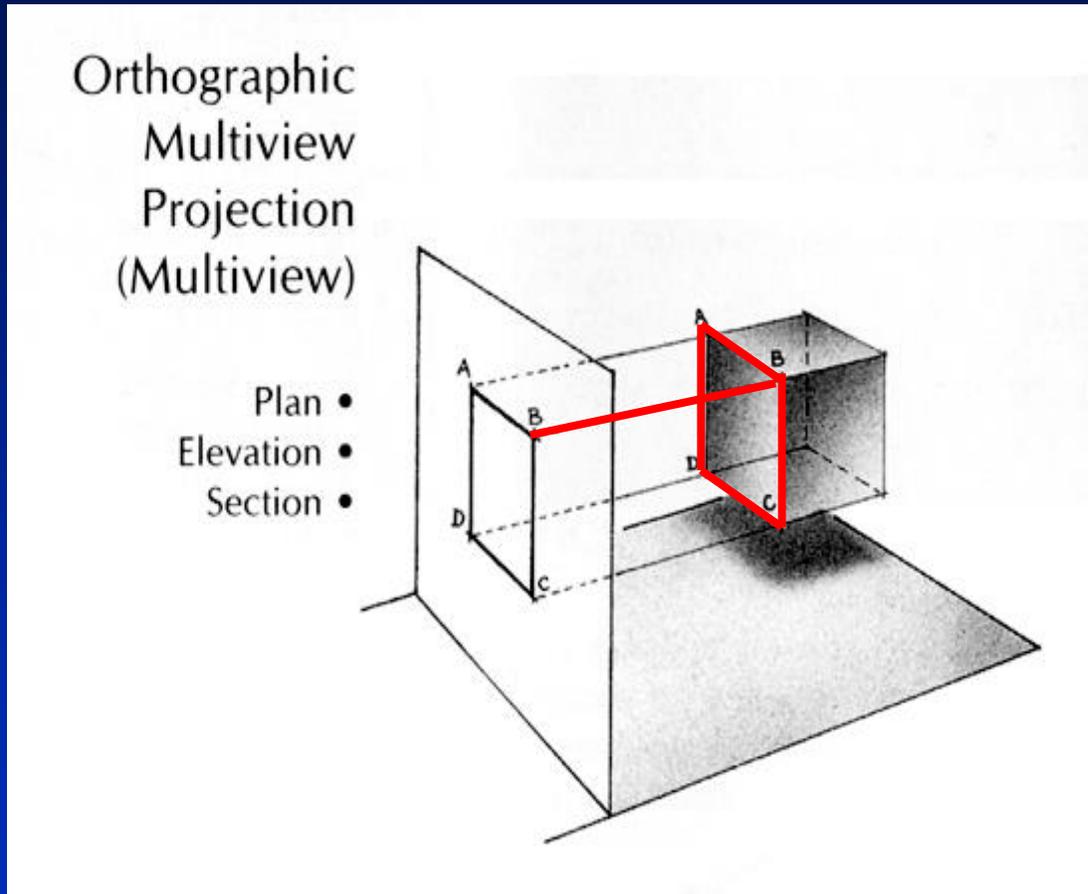
# Reference

---

- Ingrid Carlbom , Joseph Paciorek. "Planar Geometric Projections and Viewing Transformation," Computing Surveys, vol. 10, no. 4, December 1978.

(This reference contains a matrix method for combining all of the above types of projections)

# Orthographic Projections

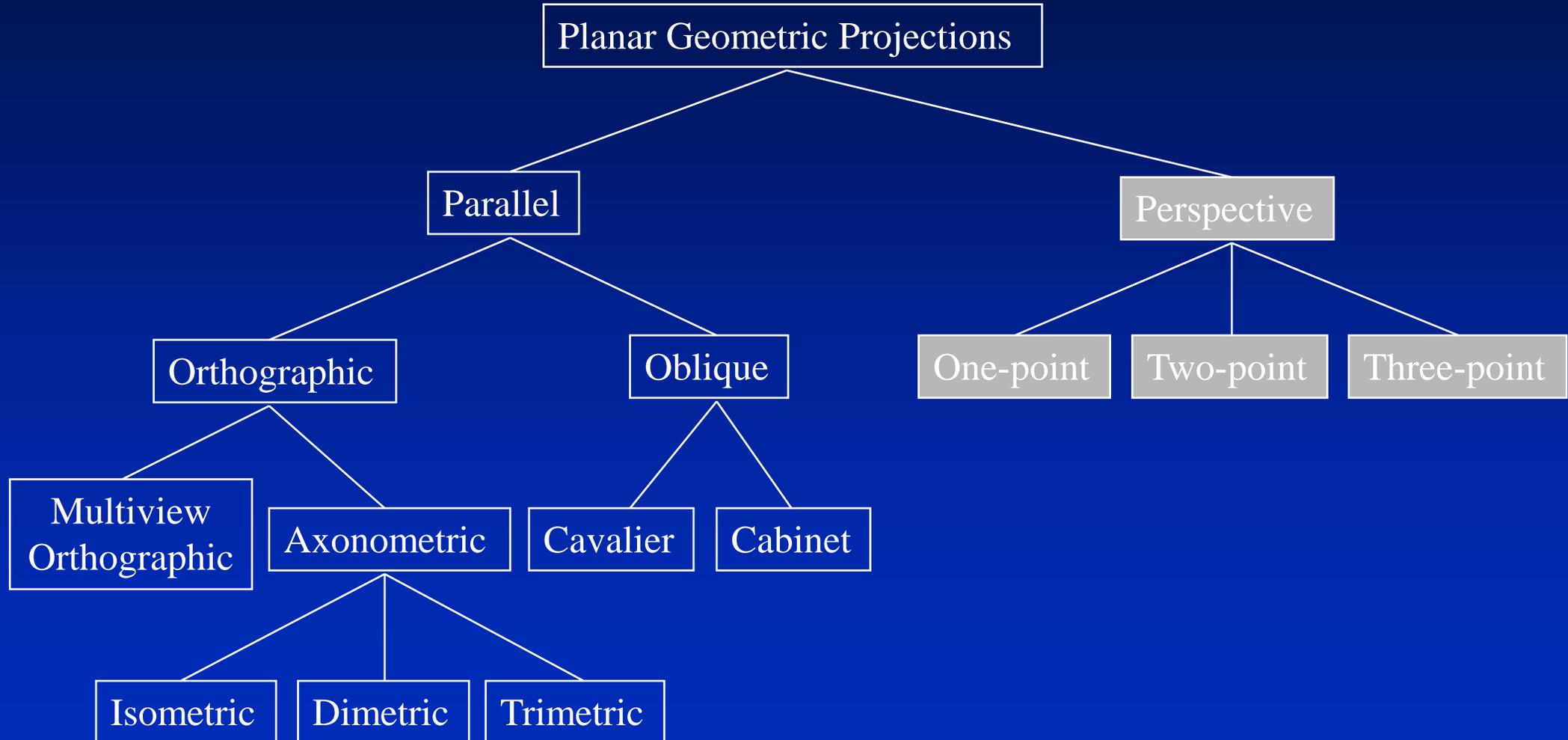


**Projectors are  
perpendicular to the  
image plane**

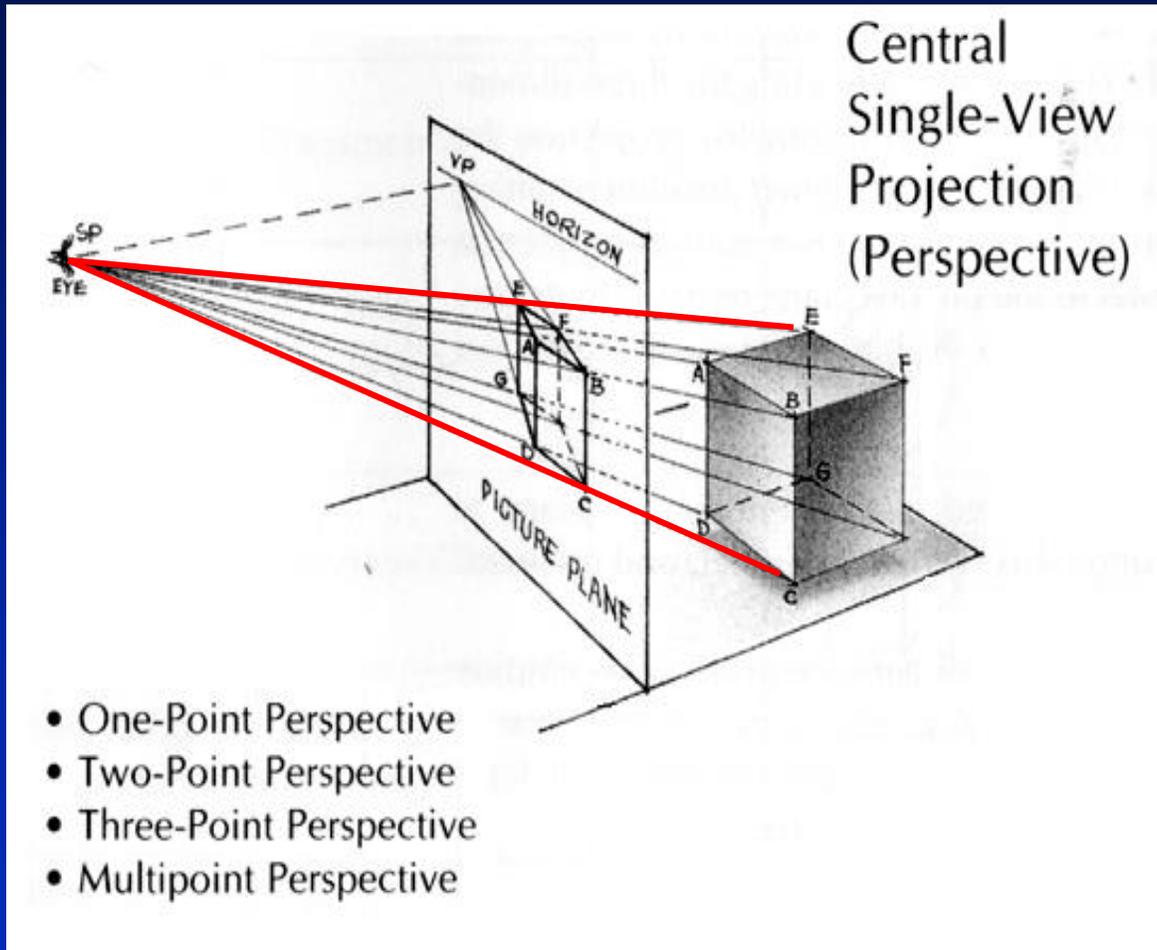
**Object faces are  
parallel to the image  
plane**

Diagram from *Axonometric and Oblique Drawing: A 3-D Construction, Rendering, and Design Guide* by M. Saleh Uddin. New York: McGraw-Hill. © 1997. P. 9.

# Planar Geometric Projections

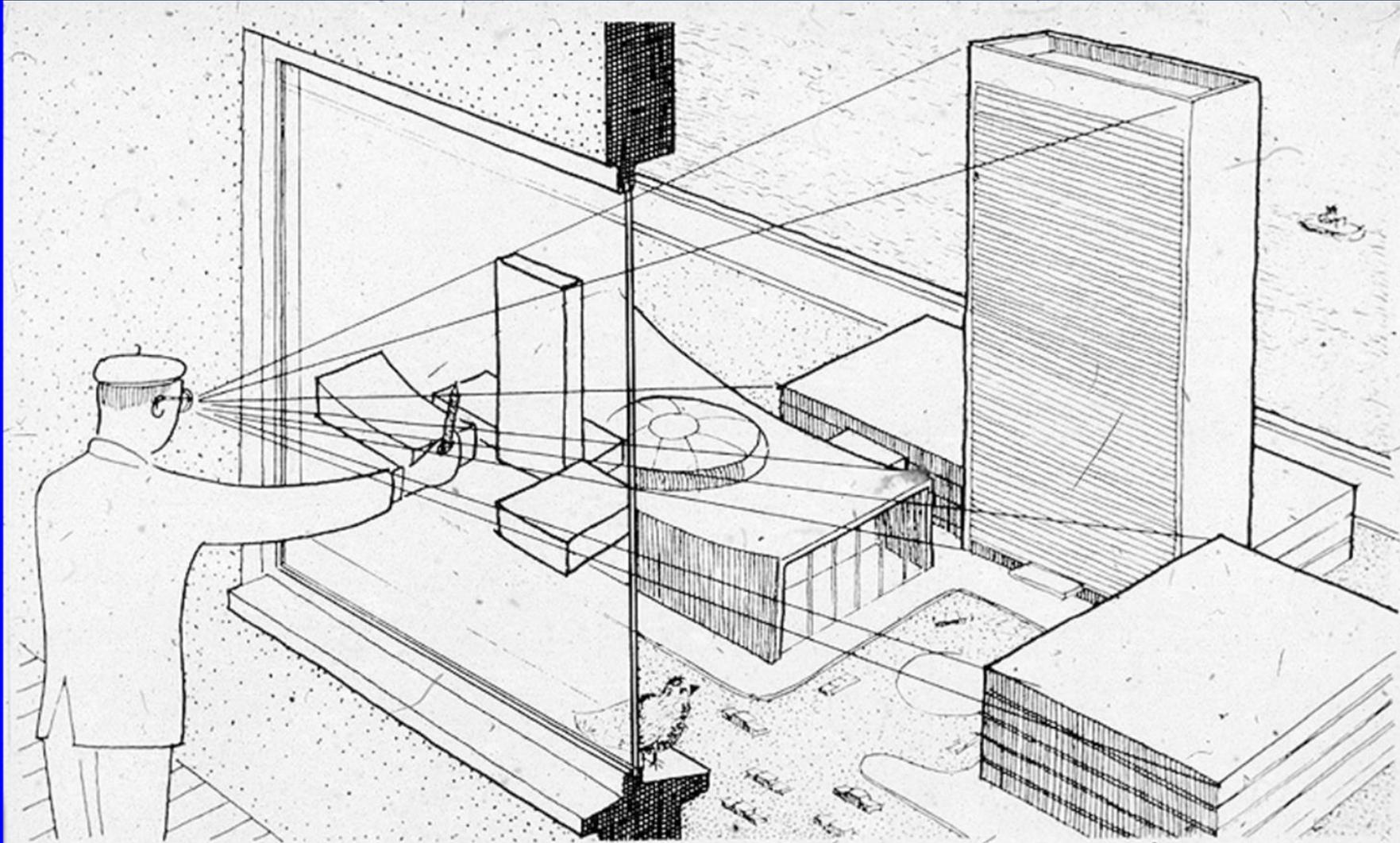


# Perspective Projection



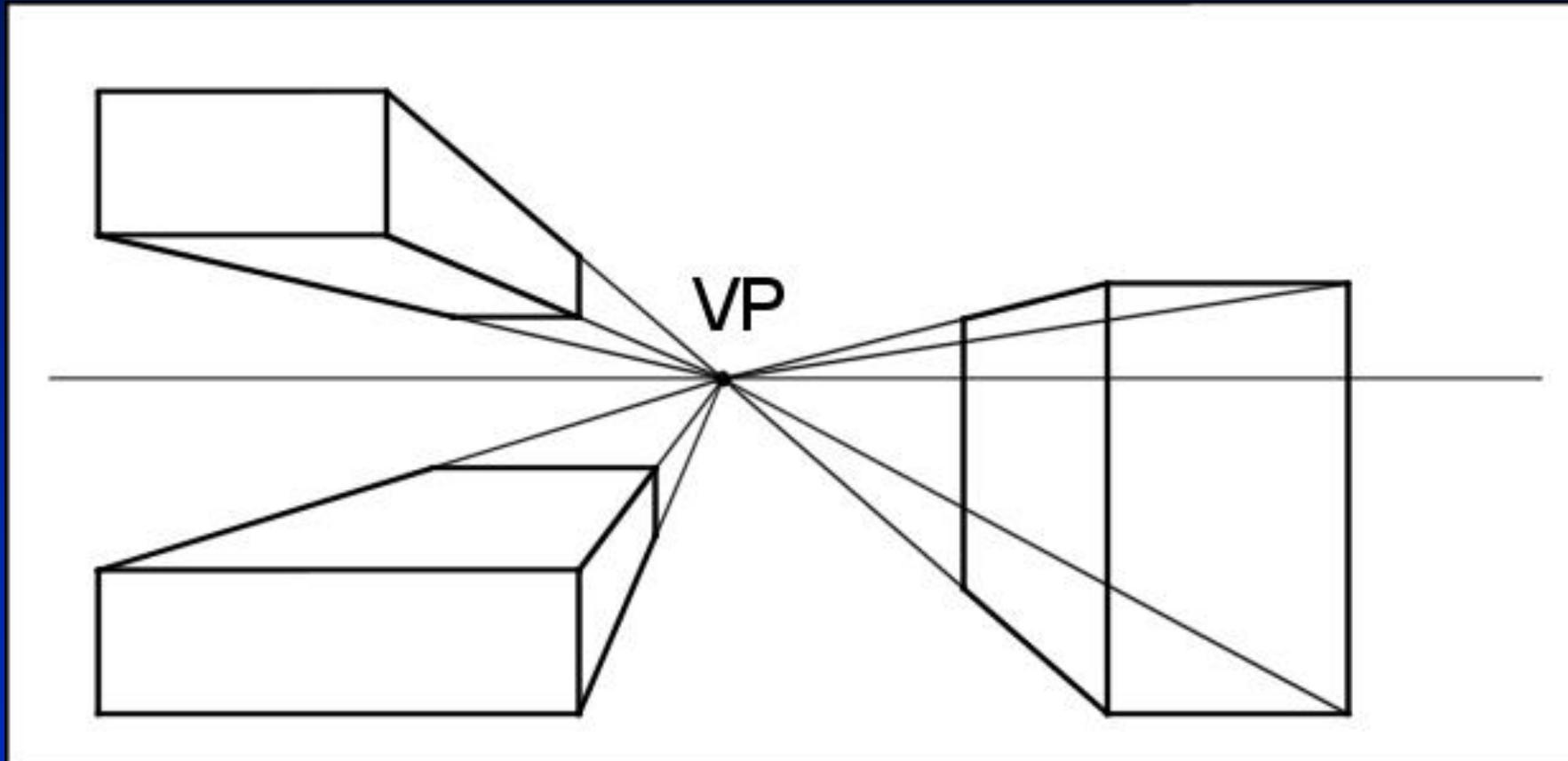
**Projectors are not parallel but converge on a single focal point (eye, camera)**

# Picture Plana Looking Through a Window

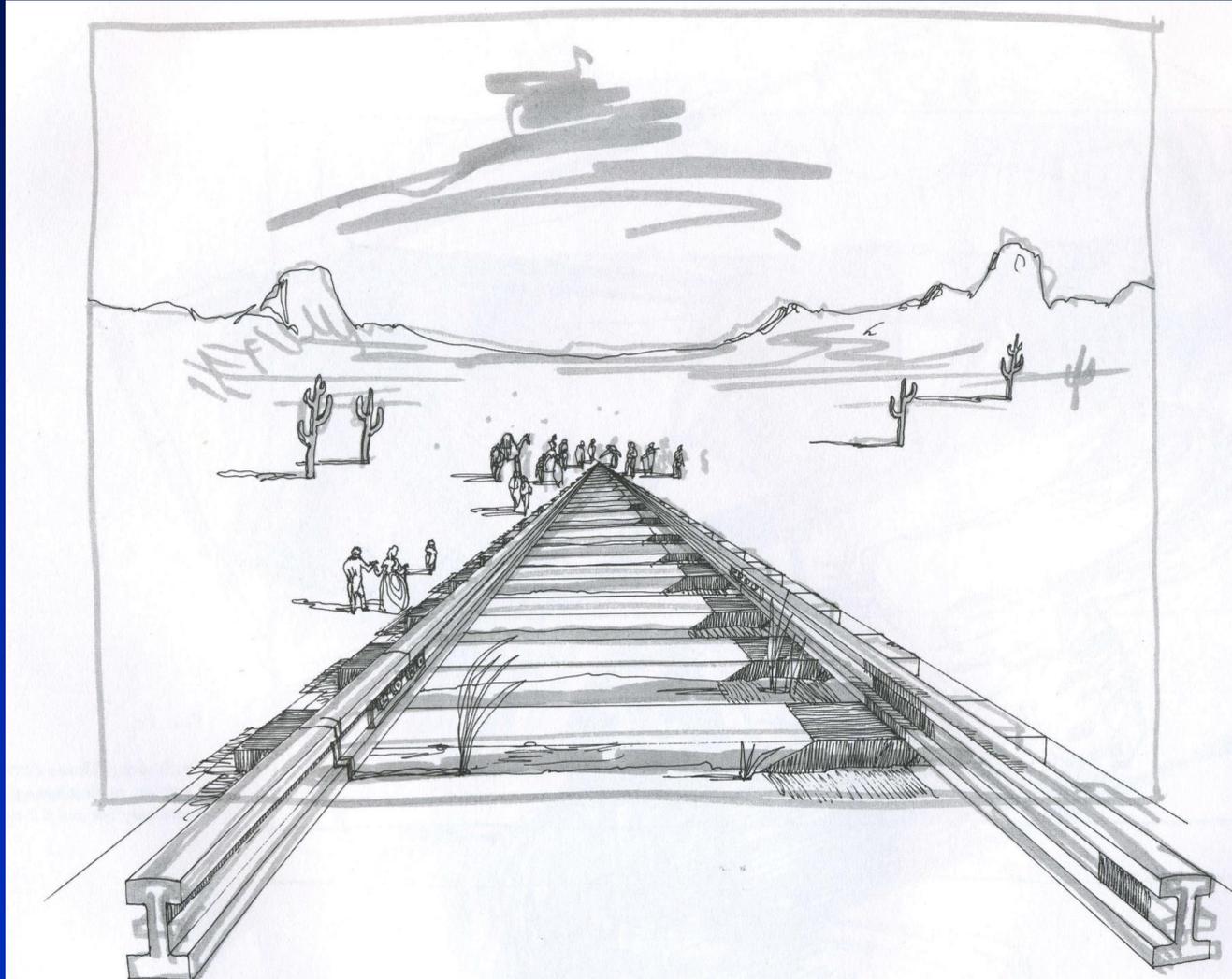


Note: all rays converge on our one cyclopean eye

# One Point Perspective



# Locating the Vanishing Point



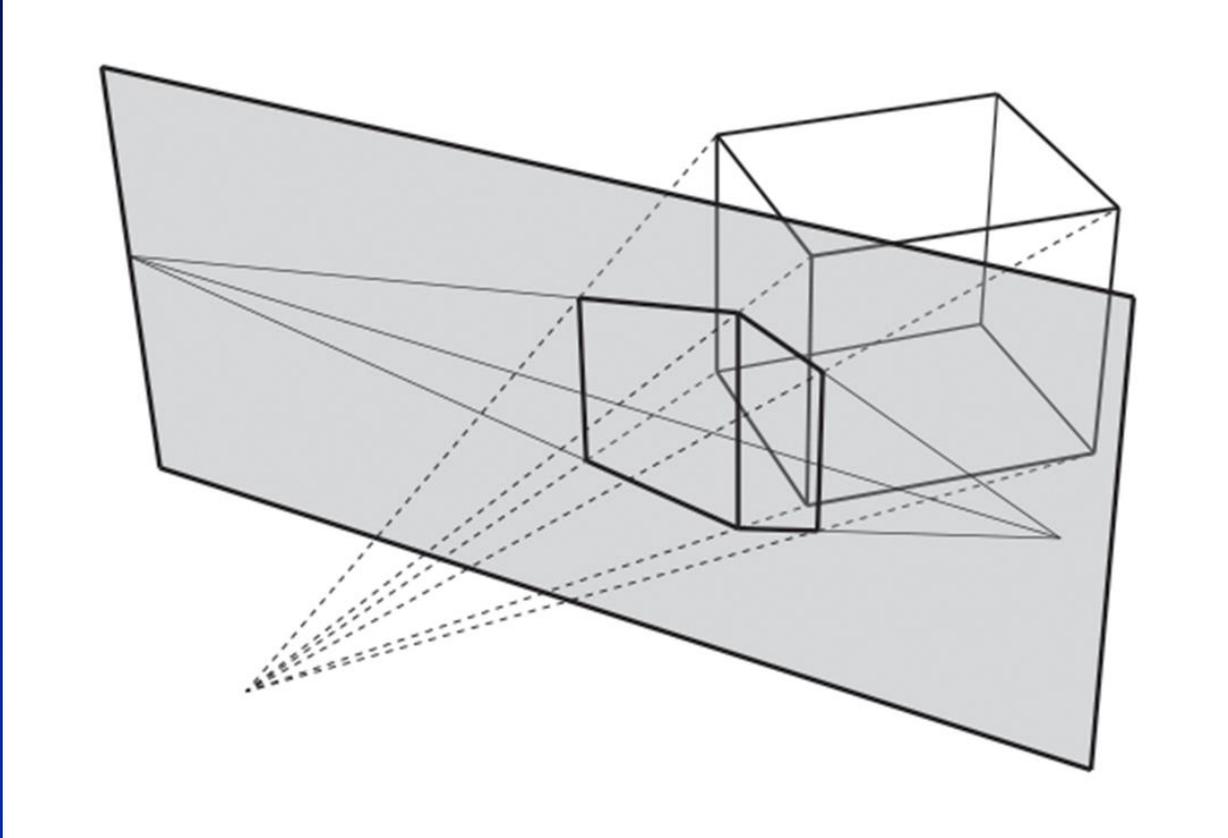
David Macauley,  
Locating the Vanishing Point

---

**What is a one-point perspective?**

**What is a two-point perspective?**

# Perspective Projection (2-point)



Rays of light travel from the object, through the picture plane, and to the viewer's eye. This is the basis for graphical perspective.

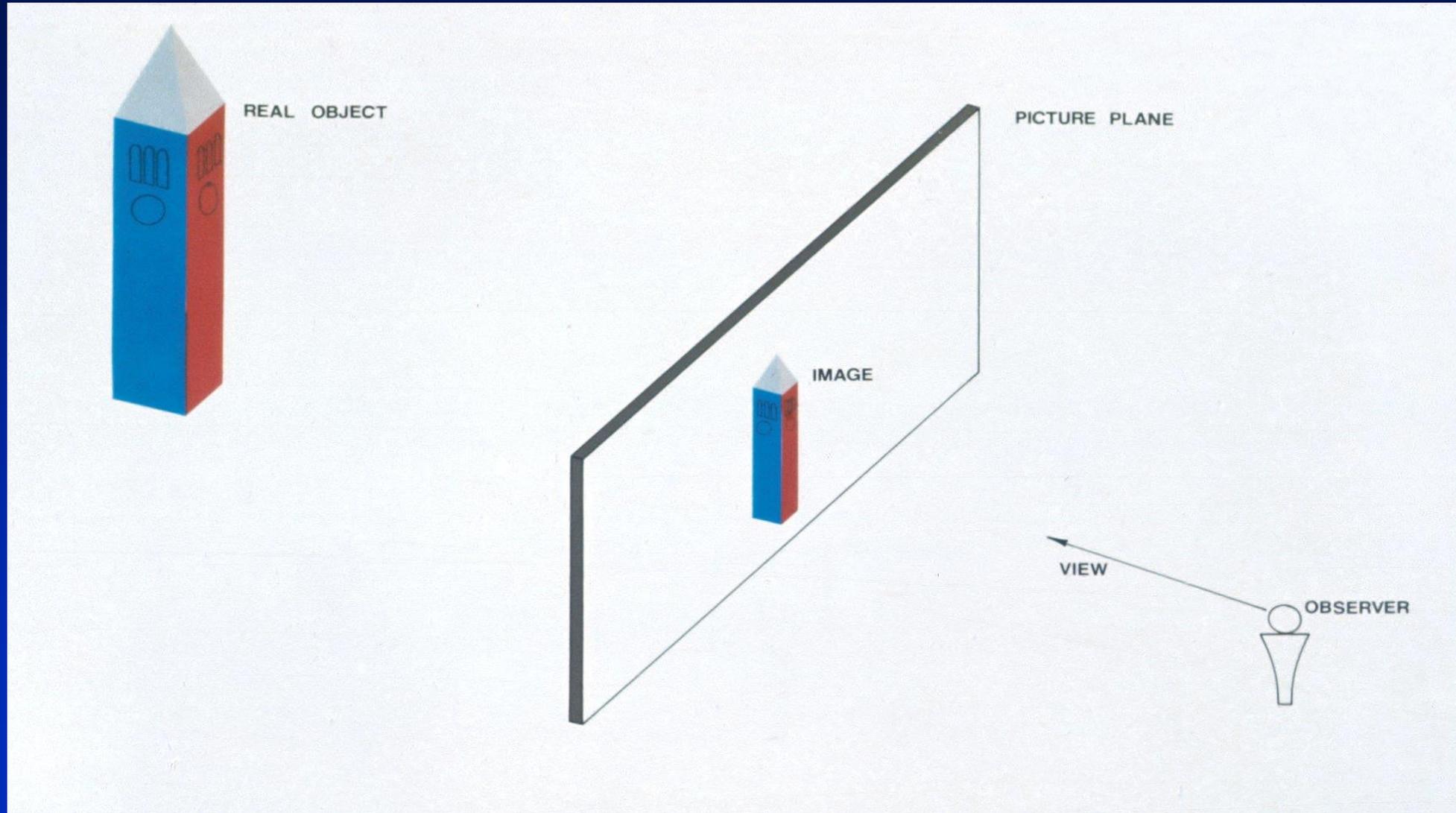
---

**What is a one-point perspective?**

**What is a two-point perspective?**

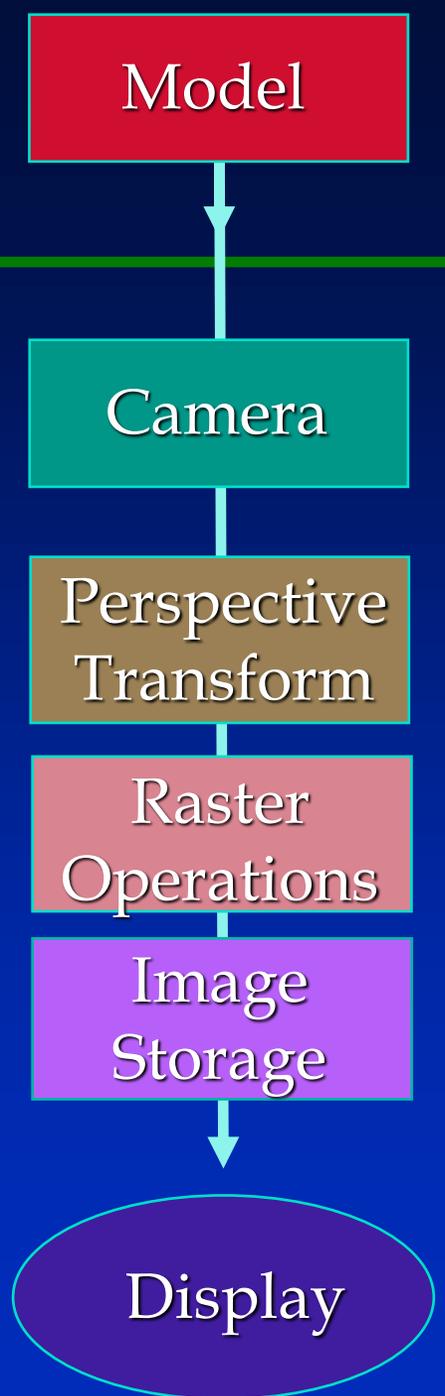
**What is a three-point perspective?**

# Computer Graphics Perspective Image Generation



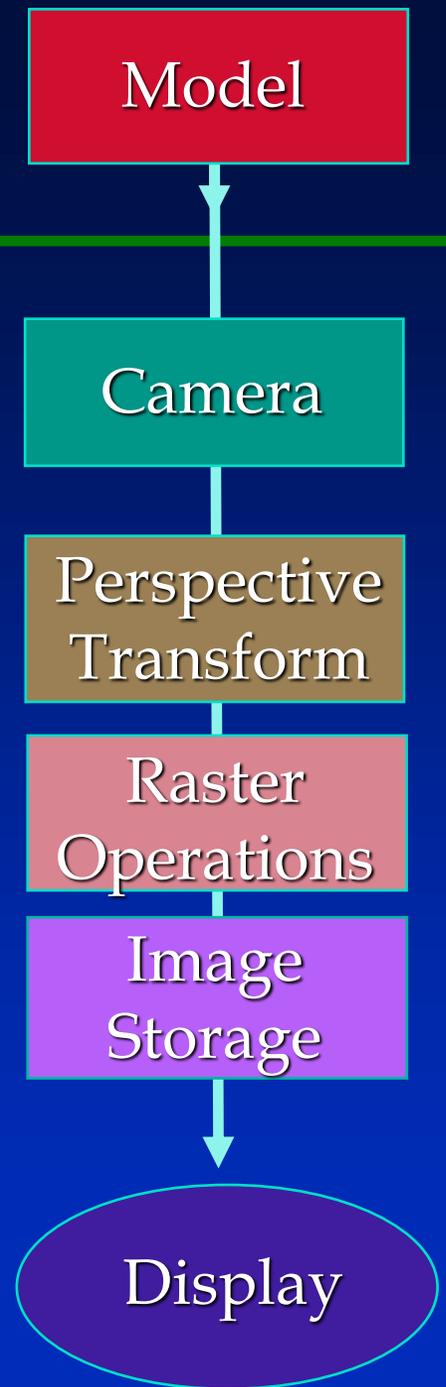
# Standard Computer Graphics Pipeline

---

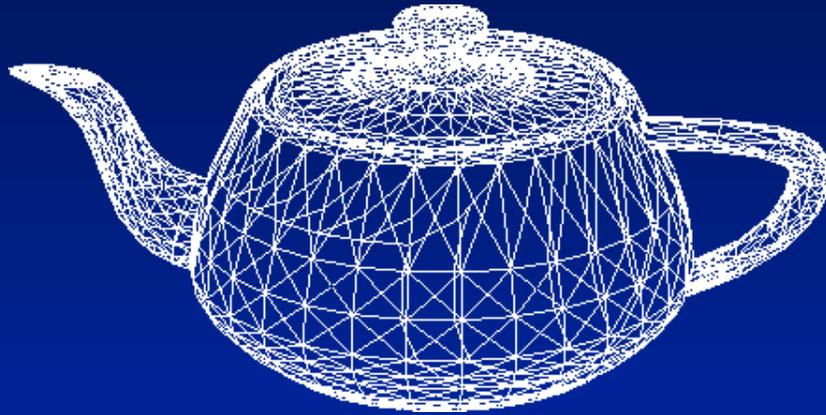


# Standard Computer Graphics Pipeline

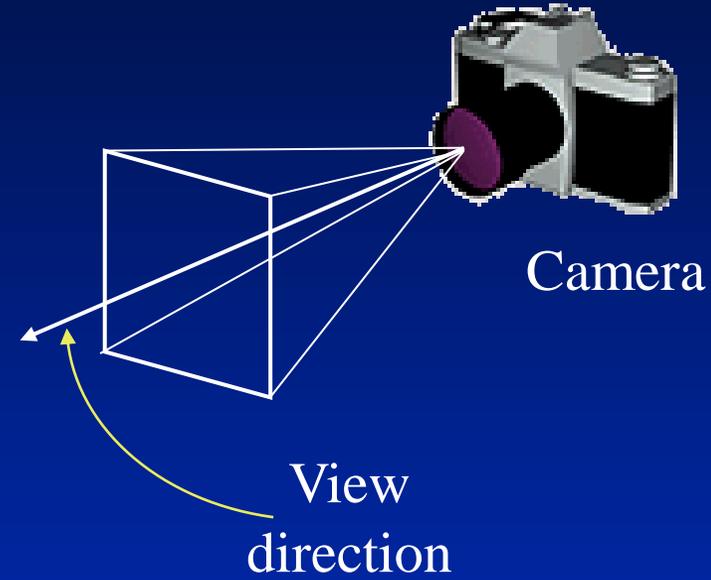
---



# Camera Definition



Model

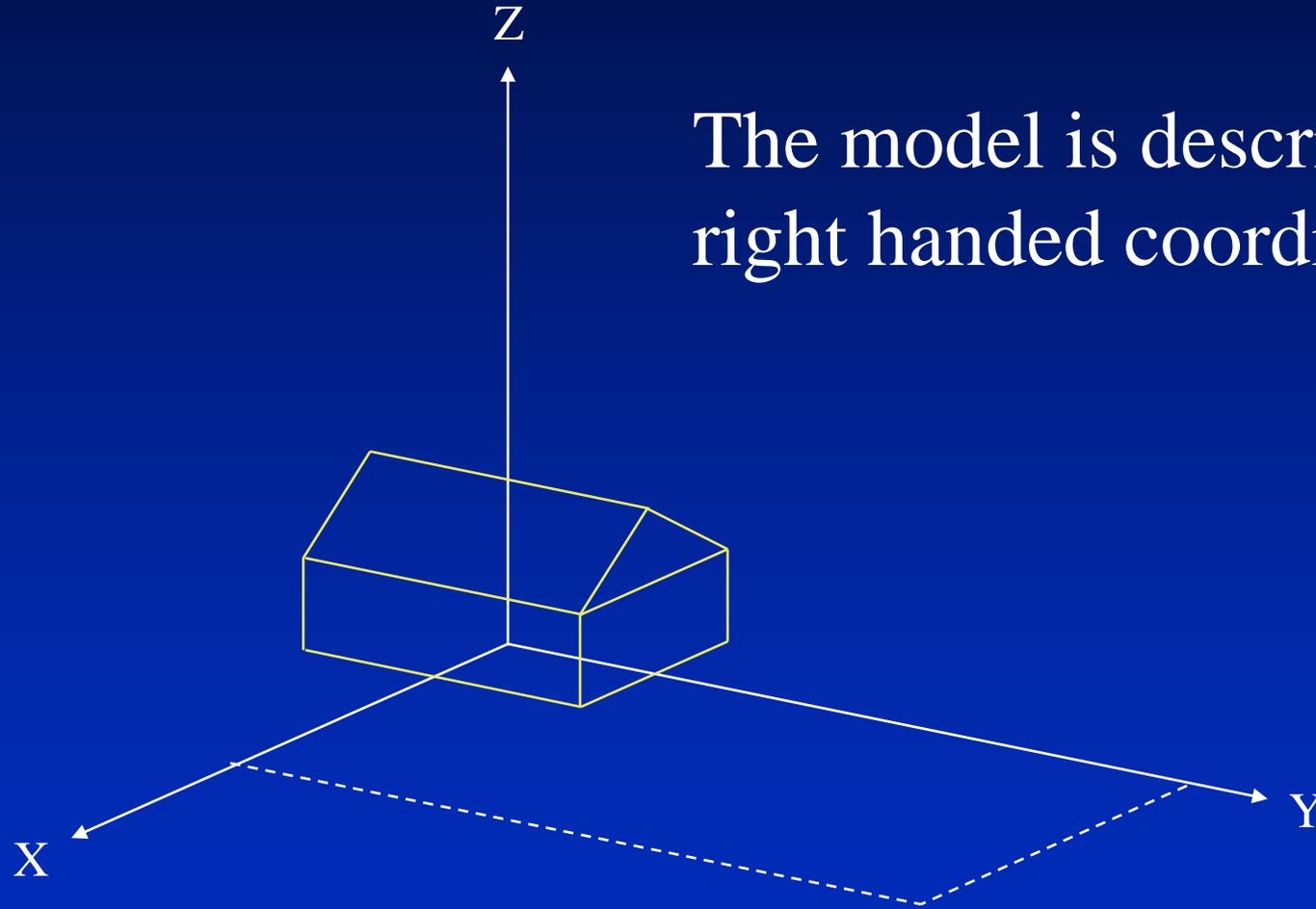


The camera location, view direction, and frustum must be defined relative to the object.

# Model Coordinate System

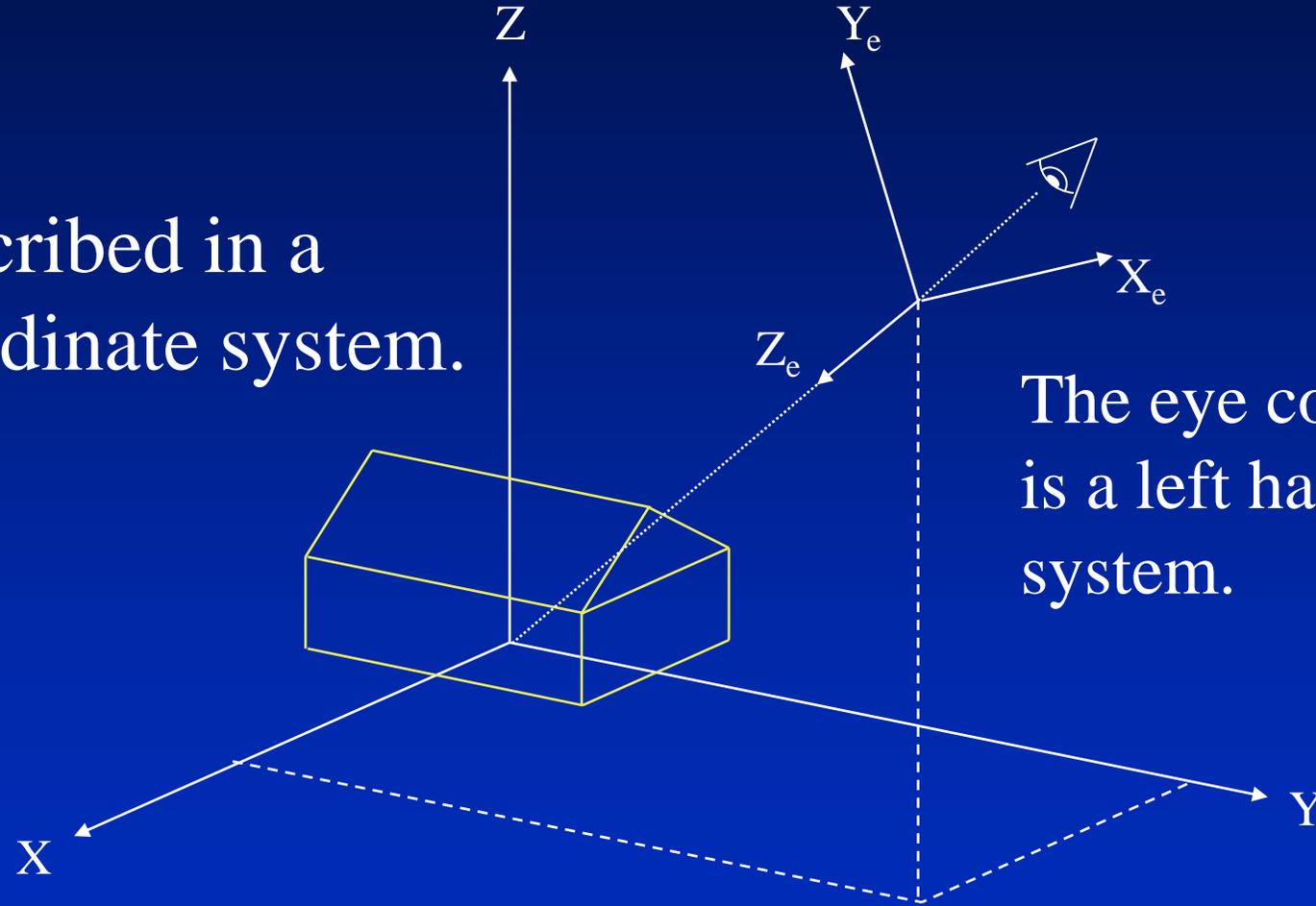
---

The model is described in a right handed coordinate system.



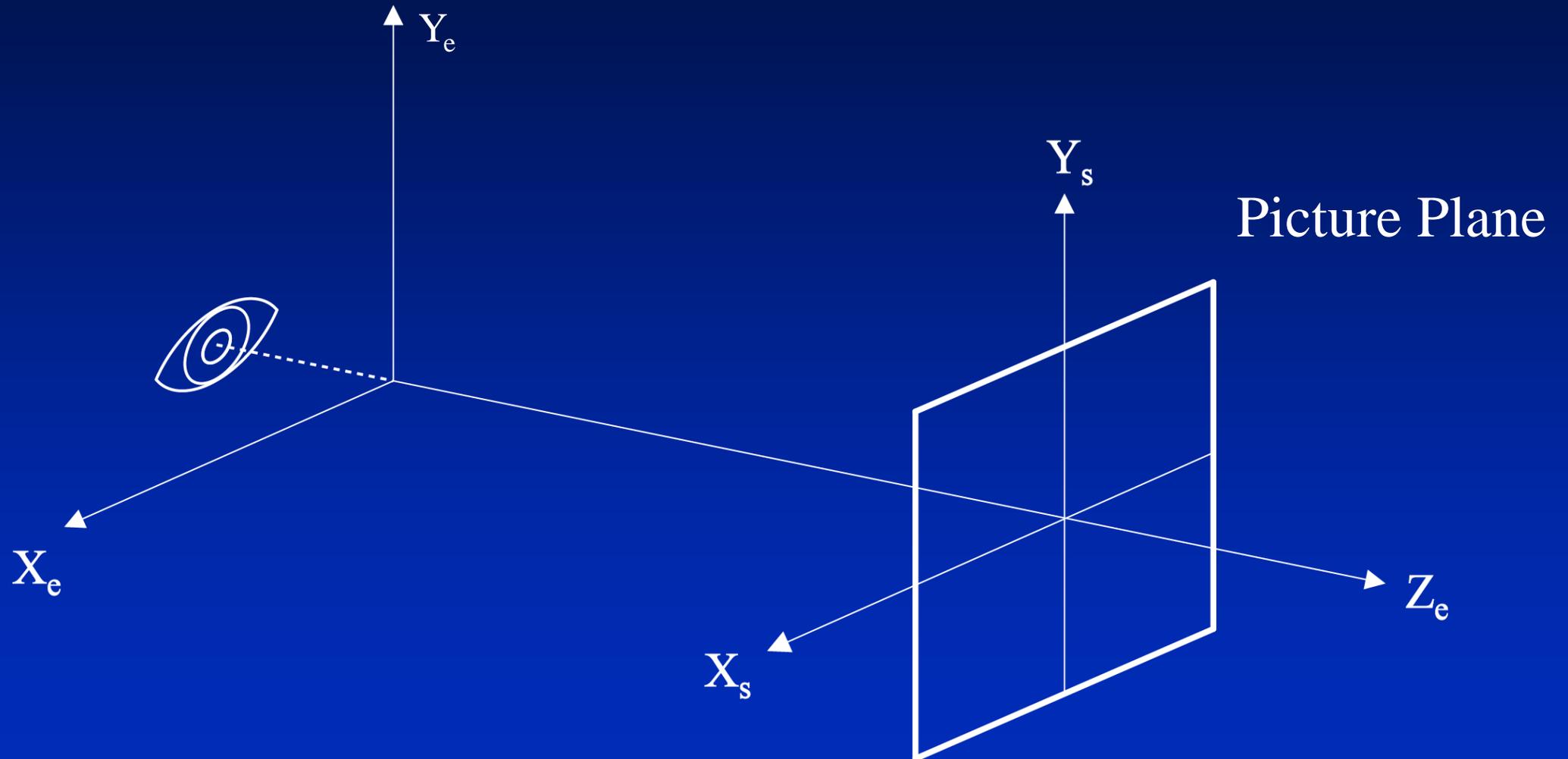
# Eye Coordinate System

The model is described in a right handed coordinate system.



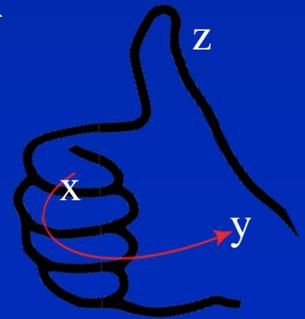
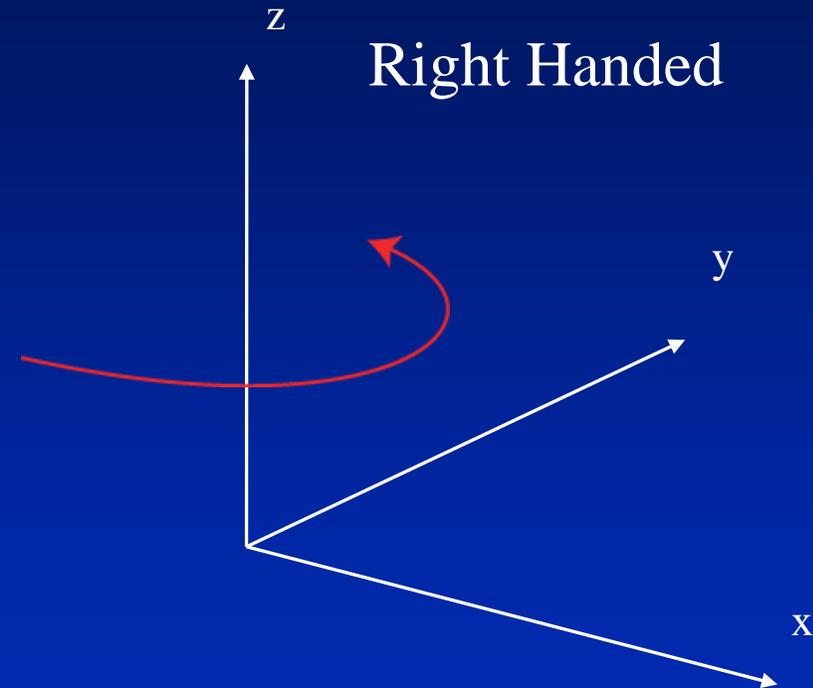
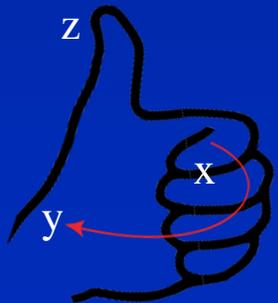
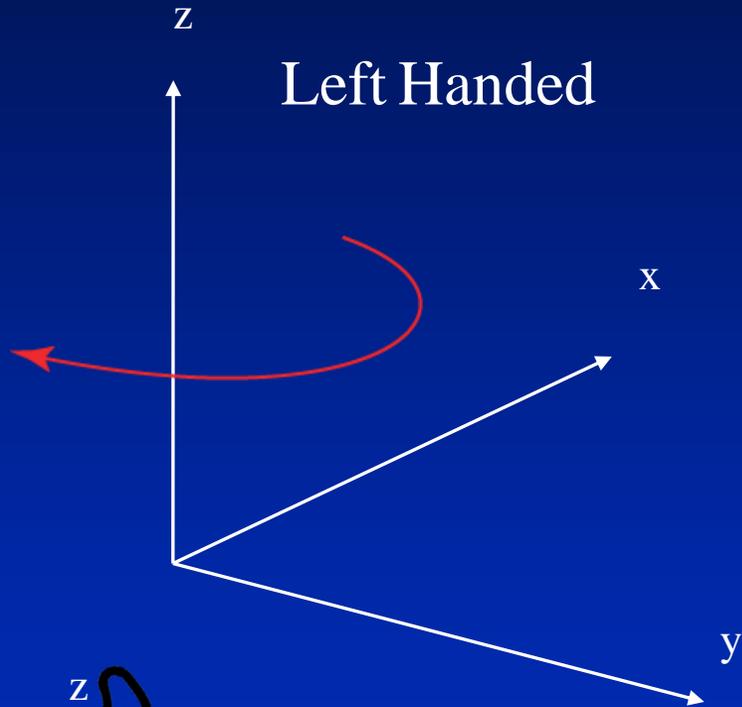
The eye coordinate system is a left handed coordinate system.

# Eye Coordinate System

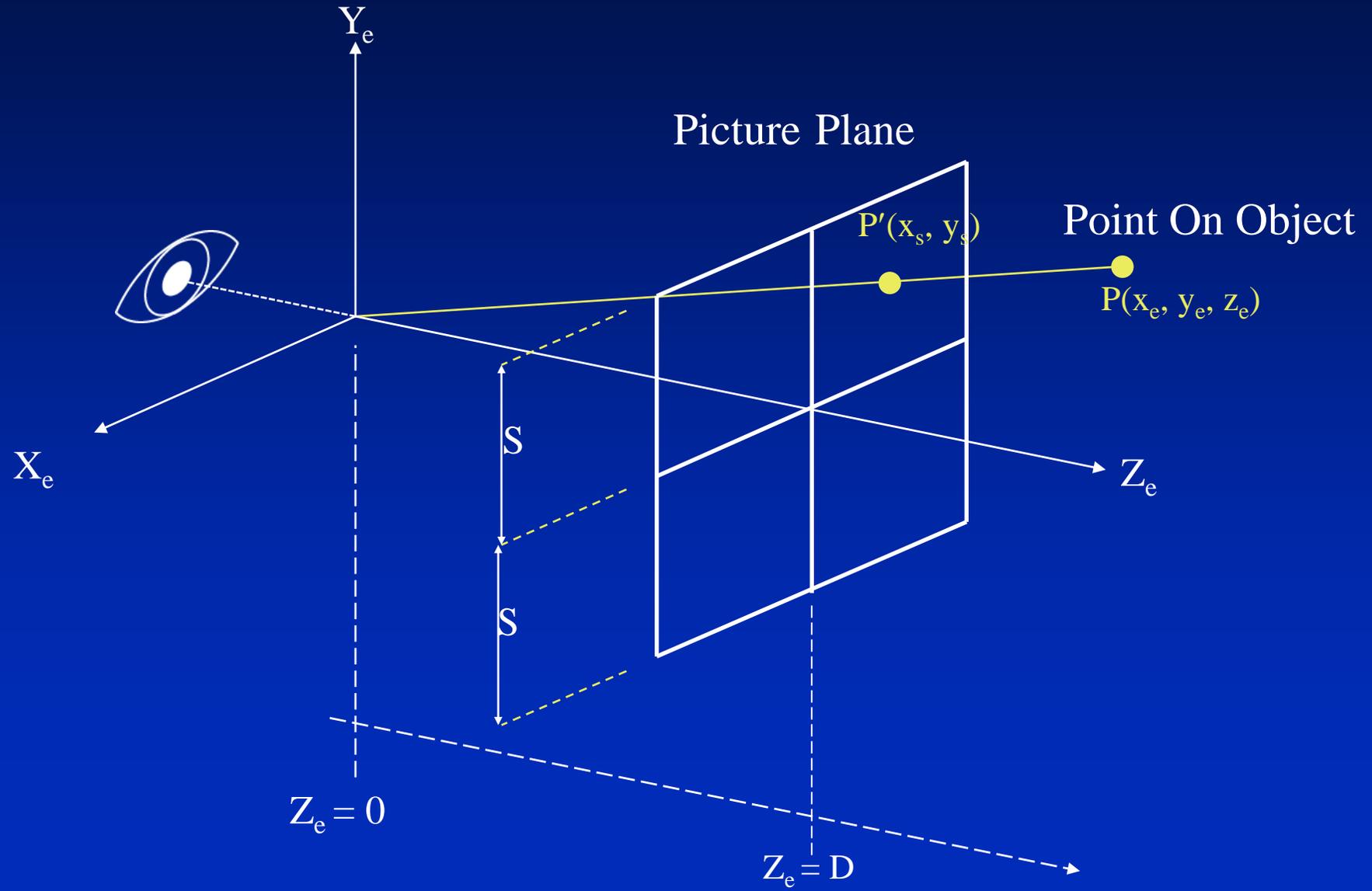


Note the eye coordinate system is a left-handed coordinate system

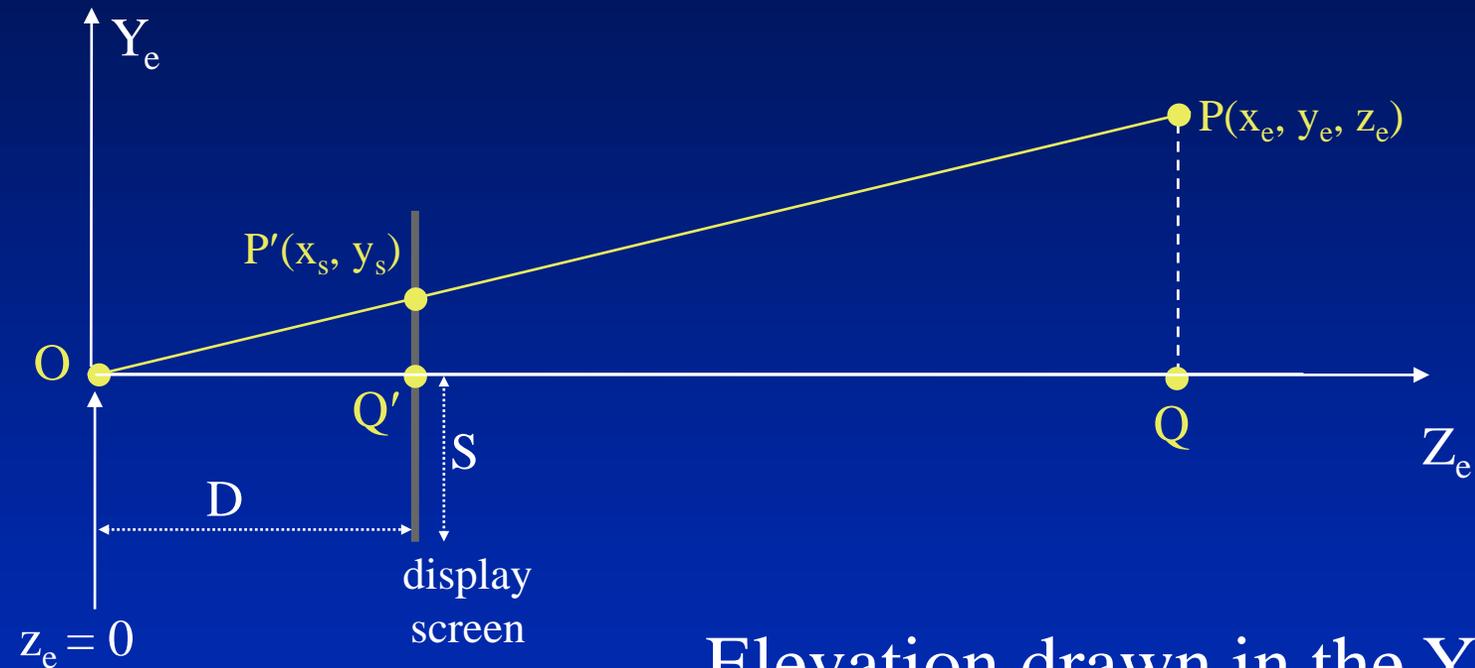
# Left Handed and Right Handed Coordinate Systems



# Simple Perspective Transformation



# Simple Perspective Transformation



Elevation drawn in the  $Y_e, Z_e$  plane.

# Simple Perspective Transformation

---

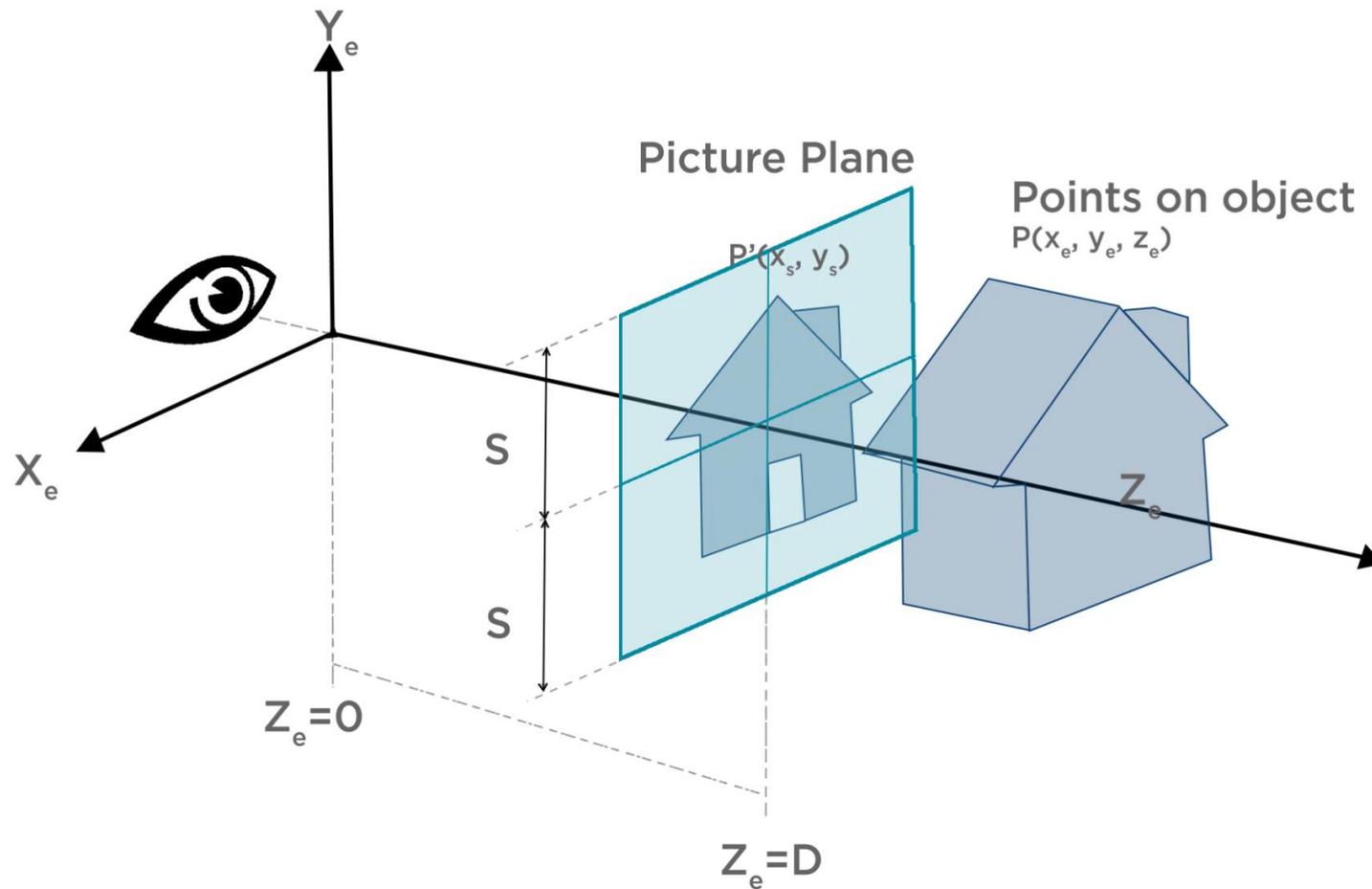
$$\frac{x_s}{D} = \frac{x_e}{z_e}, \quad \frac{y_s}{D} = \frac{y_e}{z_e}$$

$$x_s = \frac{Dx_e}{z_e}, \quad y_s = \frac{Dy_e}{z_e}$$

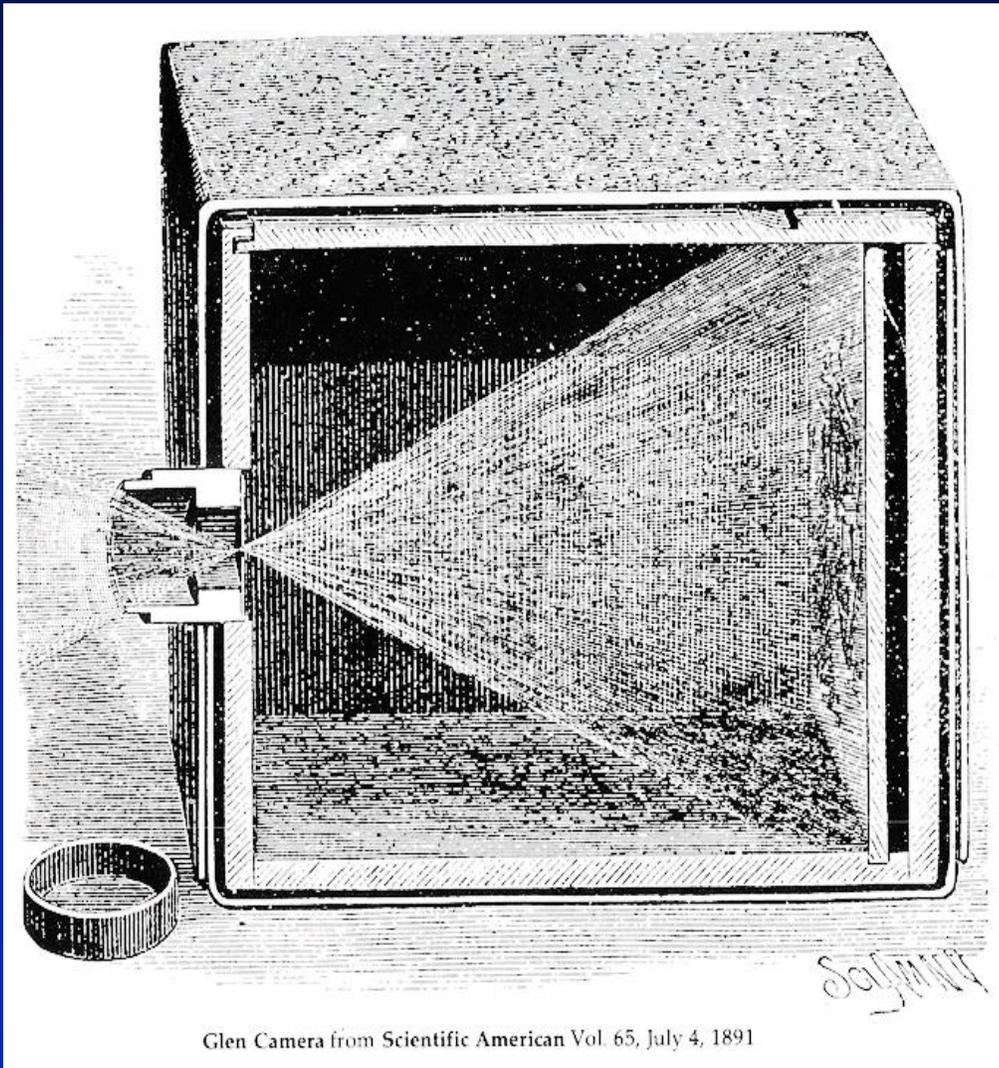
To convert to a dimensionless fraction,  
can divide by the window size  $S$ .

$$x_s = \frac{Dx_e}{Sz_e}, \quad y_s = \frac{Dy_e}{Sz_e}$$

# Transformations - video



# Pinhole Camera



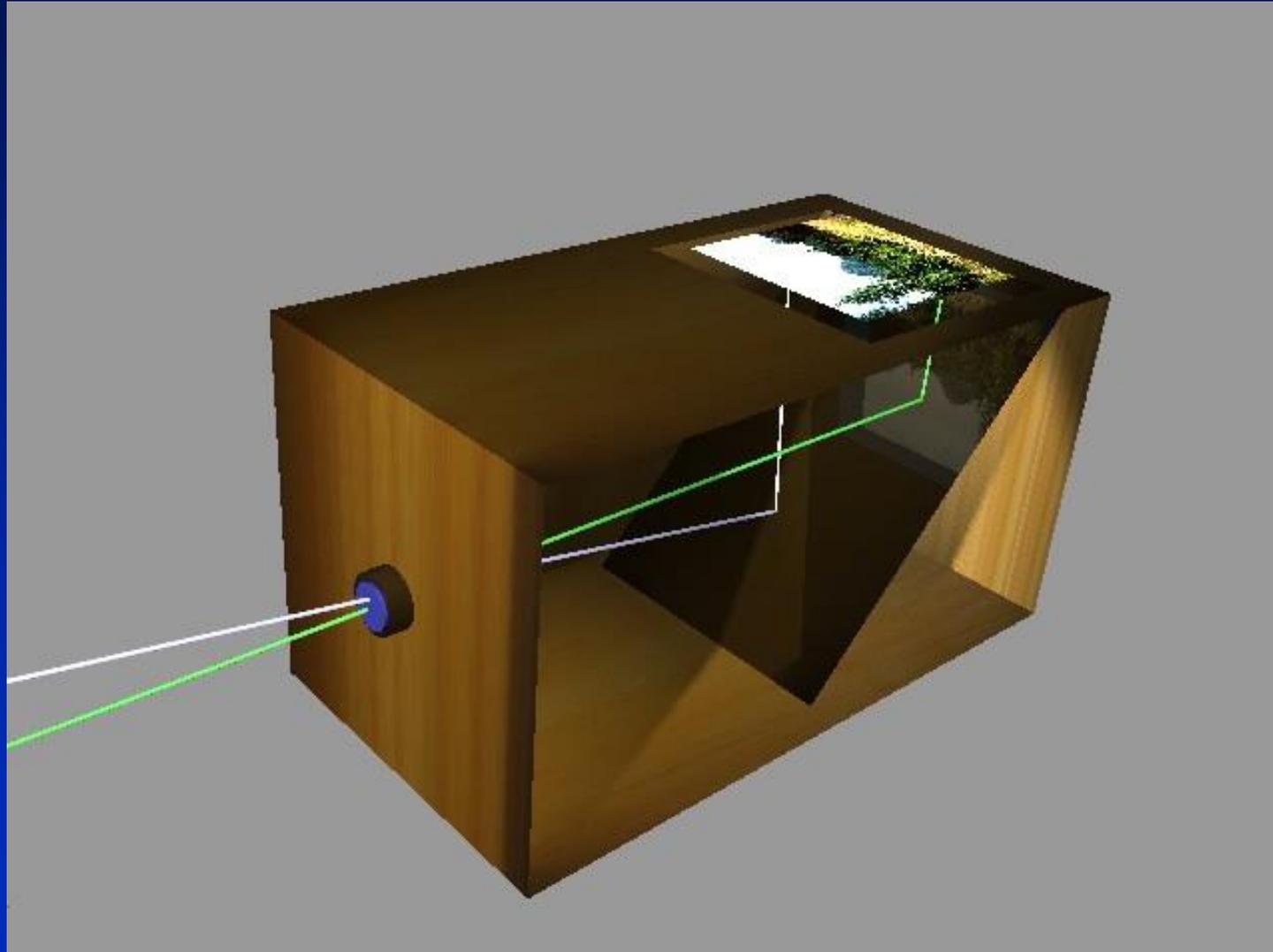
Note that the entire image through the pinhole is totally in focus on a single image plane.

# Ibn al-Haitham (Al-Hazen)



Credited with the having built the first camera obscura in the 10<sup>th</sup> Century.

# Camera Obscura



# Brunelleschi's Perspective Experiment

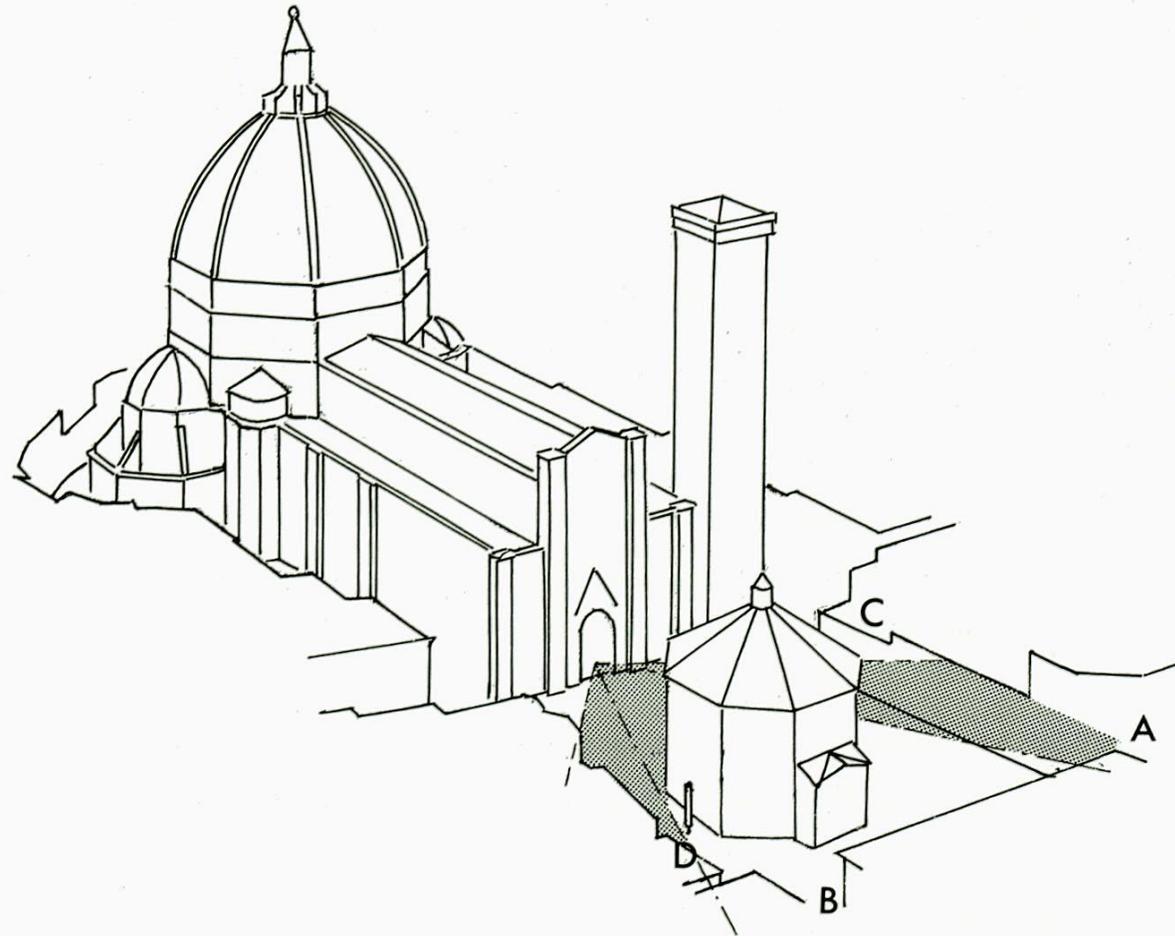
---

- How do you draw a perspective image?
- How do you know it is correct?

Martin Kemp. *THE SCIENCE OF ART*, Chapter 1 (Linear perspective from Brunelleschi to Leonardo), pp. 9-15. *AVAILABLE ON COURSE WEBSITE*

# Brunelleschi's Perspective Experiment

6 Brunelleschi's first experiment: overhead view of Florence Cathedral and the Baptistry with indication of the position of the observer inside the central portal and his two possible angles of vision.

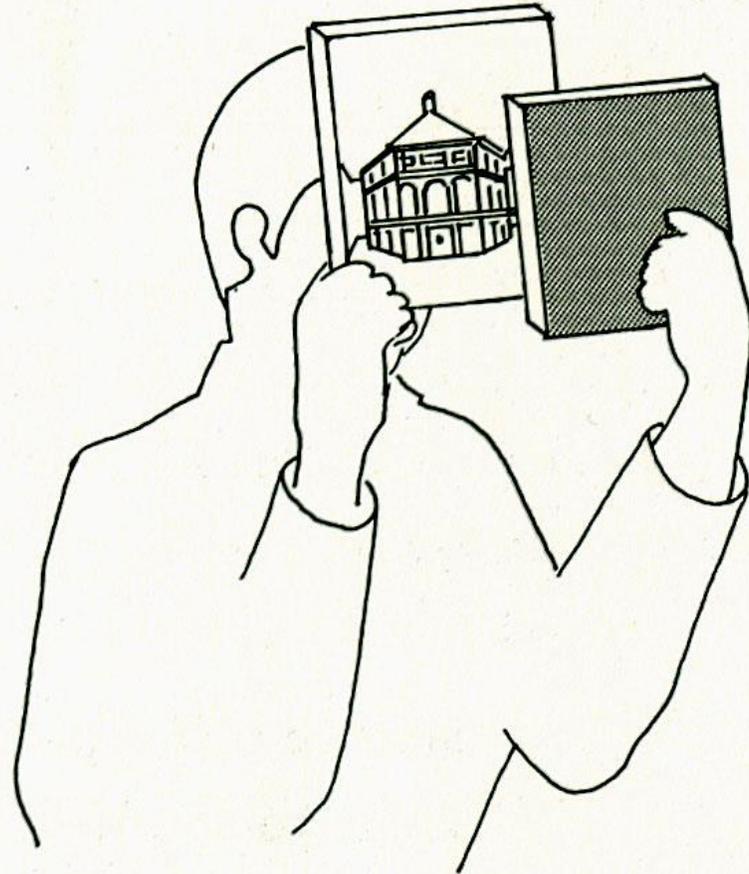


# Ghiberti's Baptistry

---



16 Brunelleschi's first  
experiment: how the  
*tavoletta* was used.



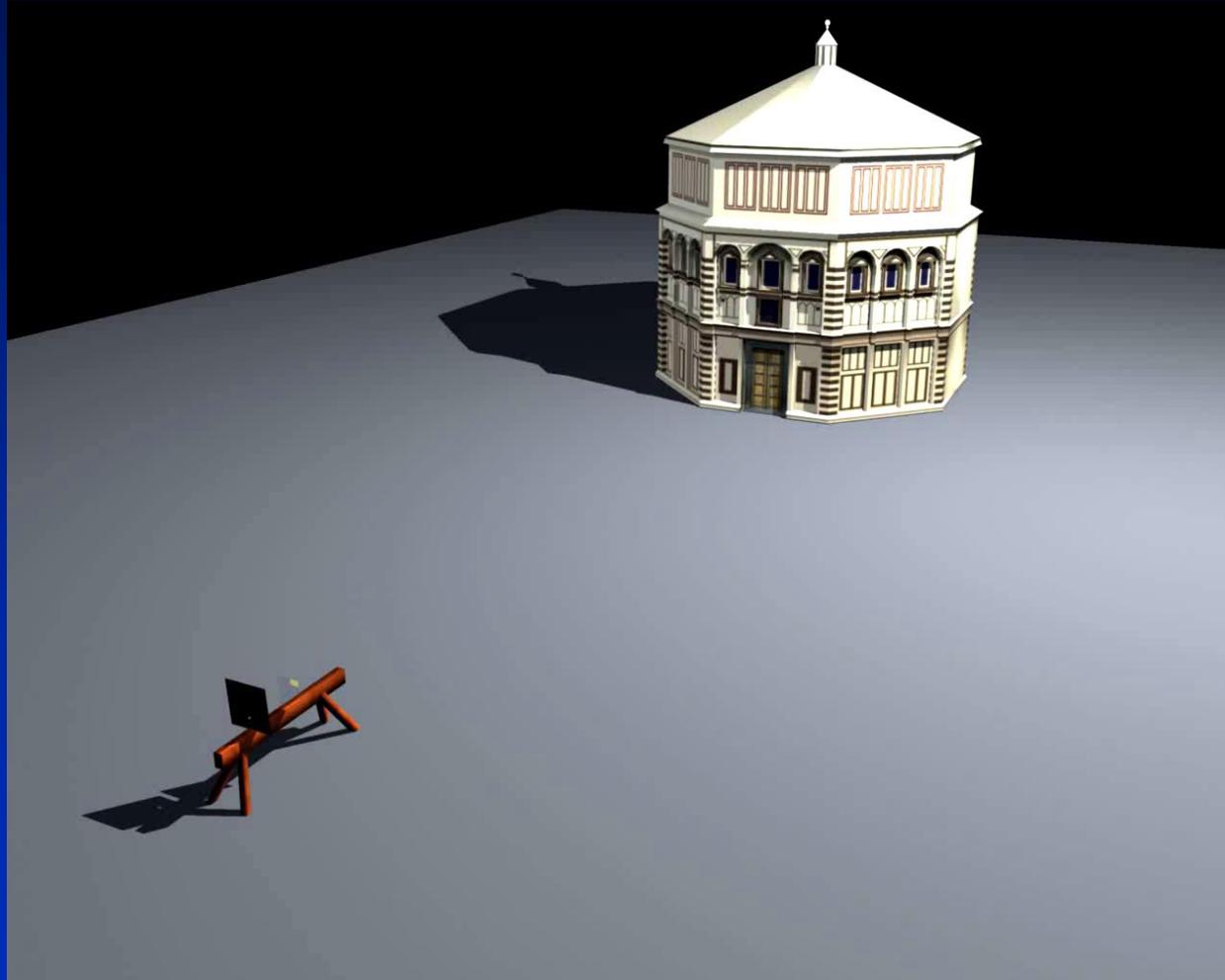
# Brunelleschi's Experiment

---



# Brunelleschi Video (render01\_wmv)

---



# Brunelleschi's Experiment



# Brunelleschi's Experiment



End. . .

---