Fast Agglomerative Clustering for Rendering

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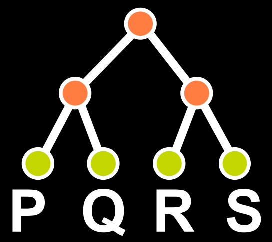
Clustering Tree

- Hierarchical data representation
  - Each node represents all elements in its subtree
  - Enables fast queries on large data
  - Tree quality = average query cost



- Examples
  - Bounding Volume Hierarchy (BVH) for ray casting
  - Light tree for Lightcuts

Tree Building Strategies

- Agglomerative (bottom-up)
  - Start with leaves and aggregate

- Divisive (top-down)
  - Start root and subdivide

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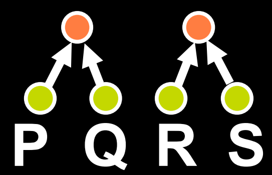

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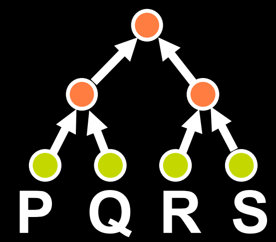

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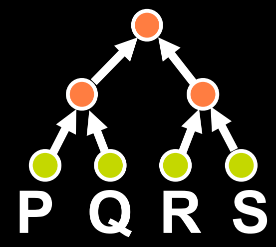

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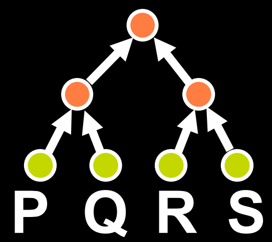

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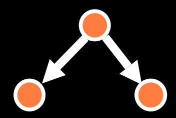

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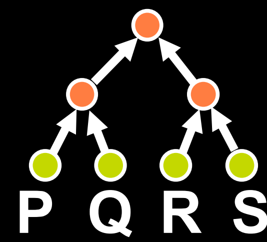

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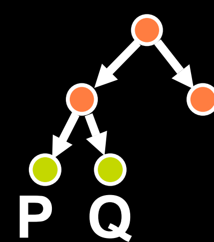

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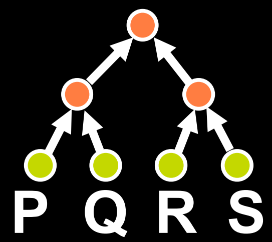

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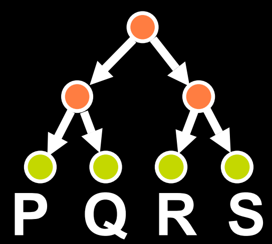


Conventional Wisdom

- Agglomerative (bottom-up)
  - Best quality and most flexible
  - Slow to build - $O(N^2)$ or worse?
- Divisive (top-down)
  - Good quality
  - Fast to build

Goal: Evaluate Agglomerative

- Is the build time prohibitively slow?
  - No, can be almost as fast as divisive
  - Much better than $O(N^2)$ using two new algorithms
- Is the tree quality superior to divisive?
  - Often yes, equal to 35% better in our tests

Related Work

- Agglomerative clustering
  - Used in many different fields including data mining, compression, and bioinformatics [eg, Olson 95, Guha et al. 95, Eisen et al. 98, Jain et al. 99, Berkhin 02]
- Bounding Volume Hierarchies (BVH)
  - [eg, Goldsmith and Salmon 87, Wald et al. 07]
- Lightcuts
  - [eg, Walter et al. 05, Walter et al. 06, Miksik 07, Akerlund et al. 07, Herzog et al. 08]

Overview

- How to implement agglomerative clustering
  - Naive $O(N^3)$ algorithm
  - Heap-based algorithm
  - Locally-ordered algorithm
- Evaluating agglomerative clustering
  - Bounding volume hierarchies
  - Lightcuts
- Conclusion

Agglomerative Basics

- Inputs
  - N elements
  - Dissimilarity function, d(A,B)
- Definitions
  - A cluster is a set of elements
  - Active cluster is one that is not yet part of a larger cluster
- Greedy Algorithm
  - Combine two most similar active clusters and repeat

Dissimilarity Function

- d(A,B): pairs of clusters –> real number
  - Measures “cost” of combining two clusters
  - Assumed symmetric but otherwise arbitrary
  - Simple examples:
    - Maximum distance between elements in A+B
    - Volume of convex hull of A+B
    - Distance between centroids of A and B

Naive $O(N^3)$ Algorithm

Repeat {

Evaluate all possible active cluster pairs <A,B>

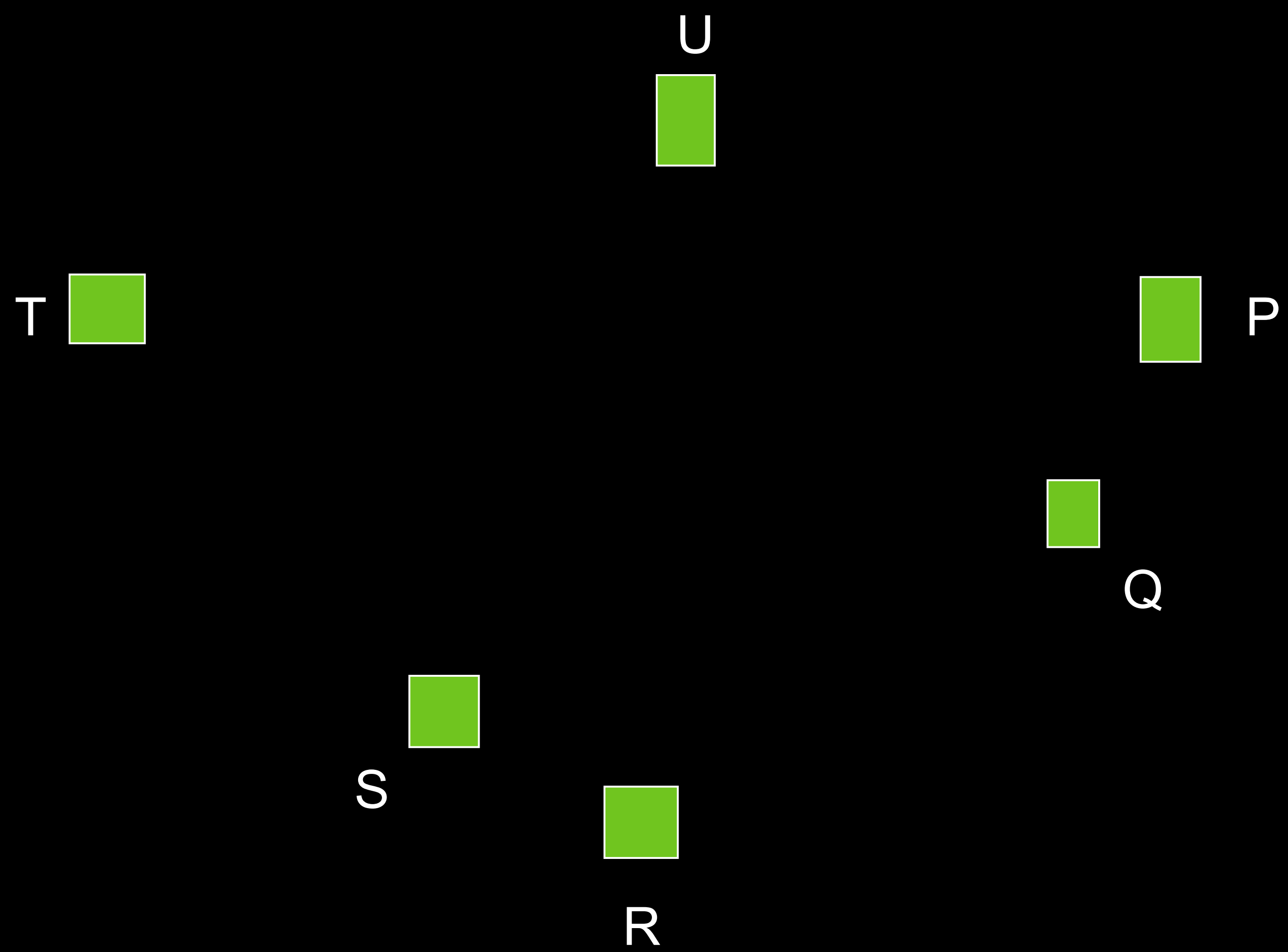
Select one with smallest d(A,B) value

Create new cluster C = A+B

} until only one active cluster left

- Simple to write but very inefficient!

Naive $O(N^3)$ Algorithm Example



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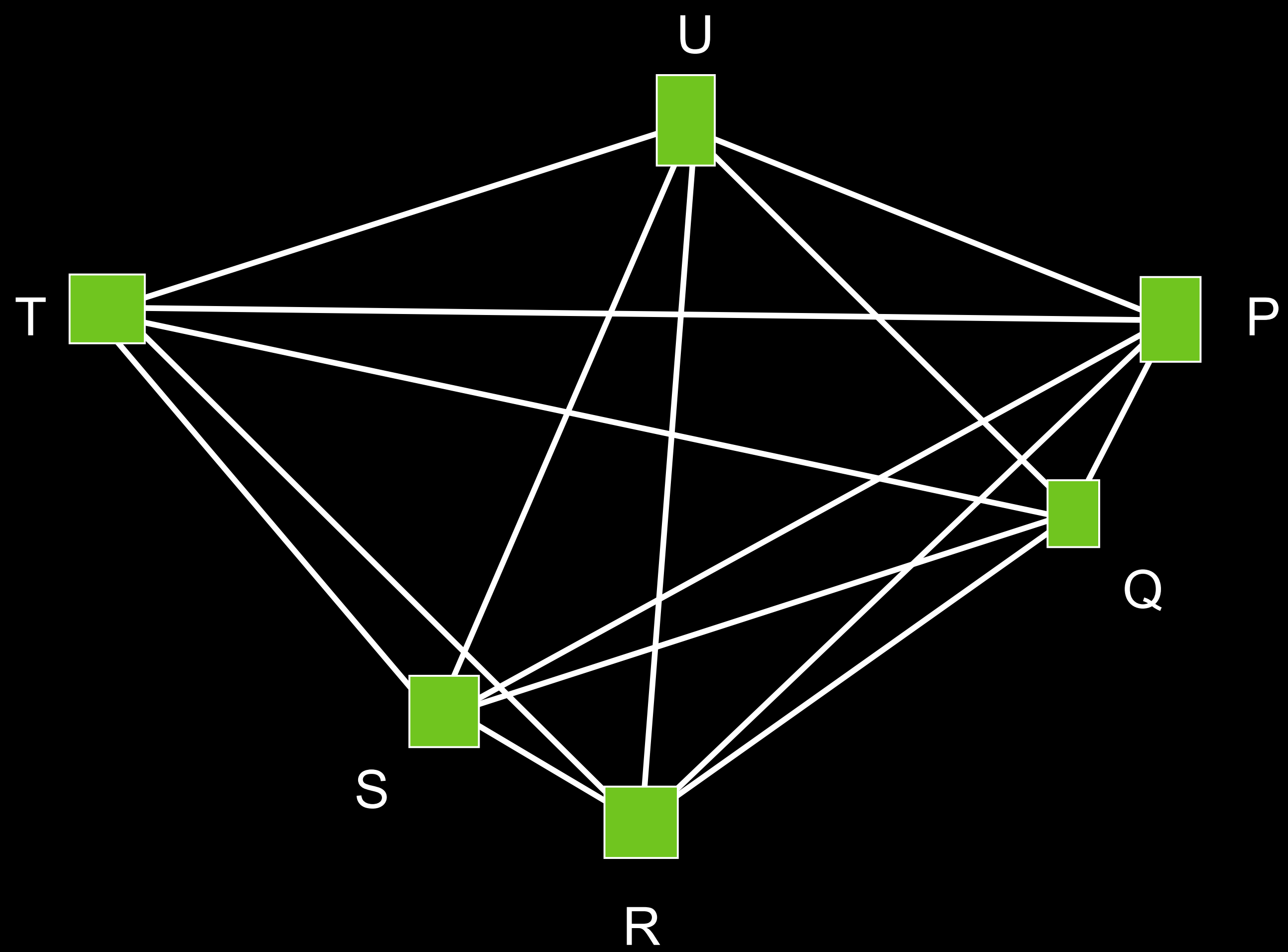

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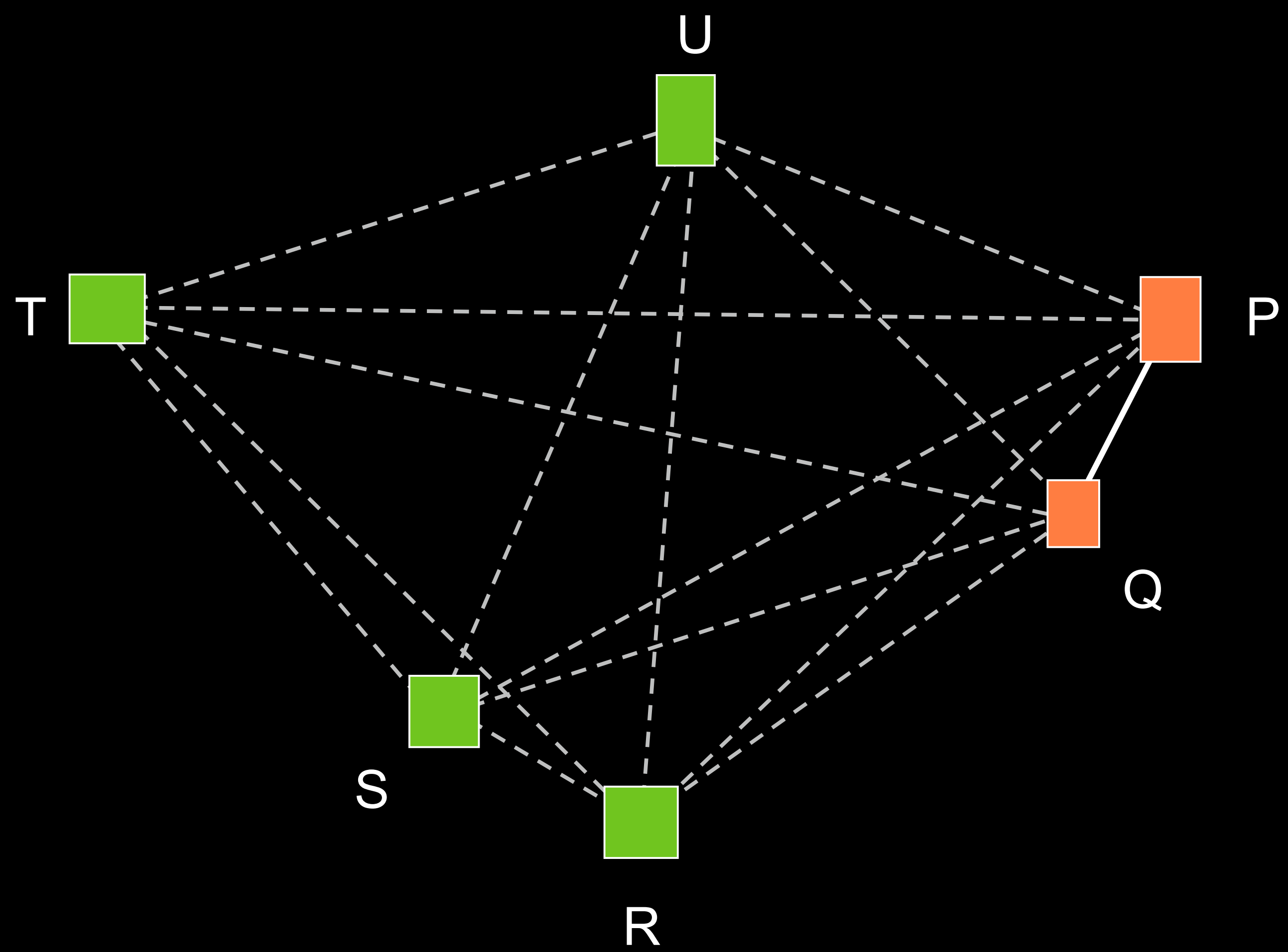

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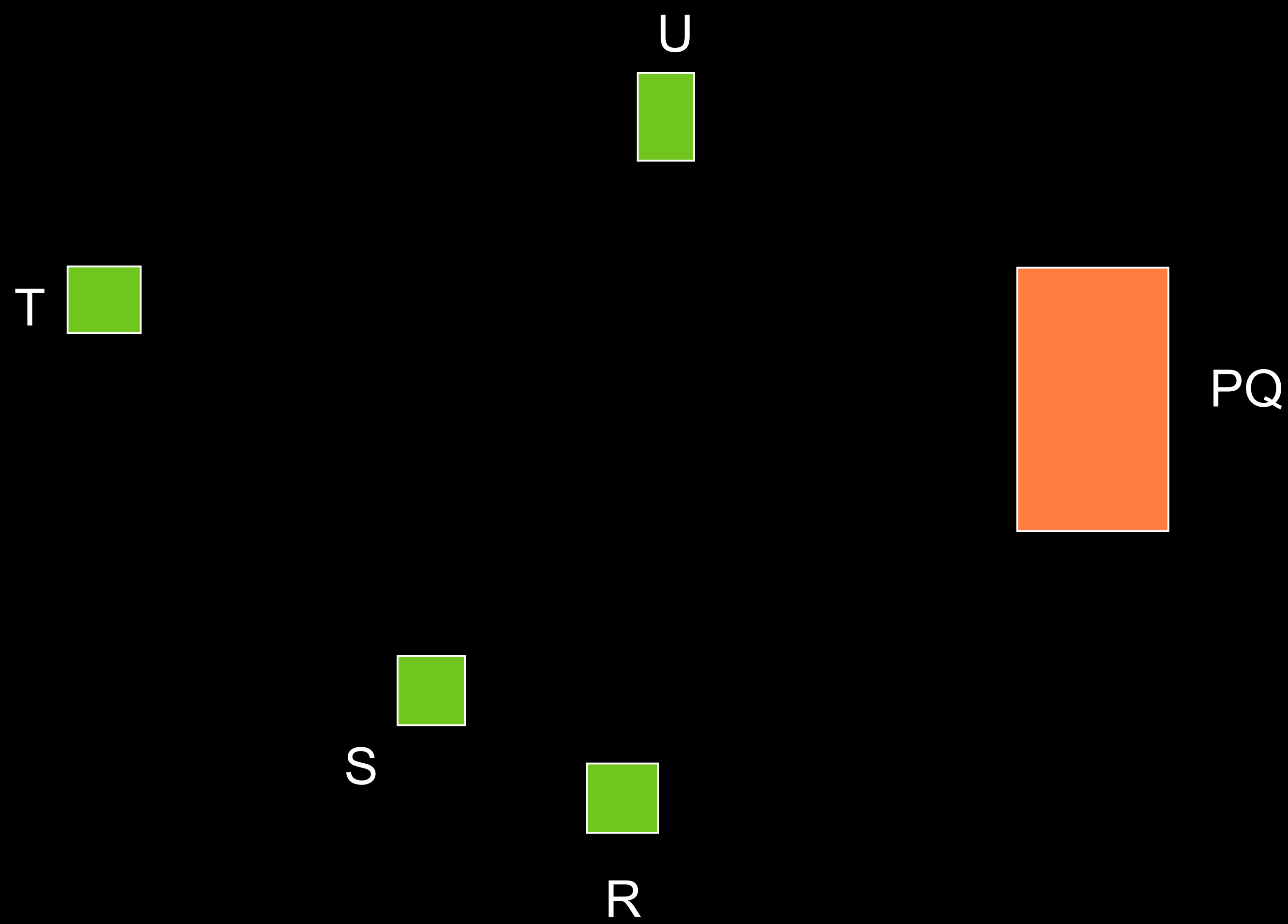

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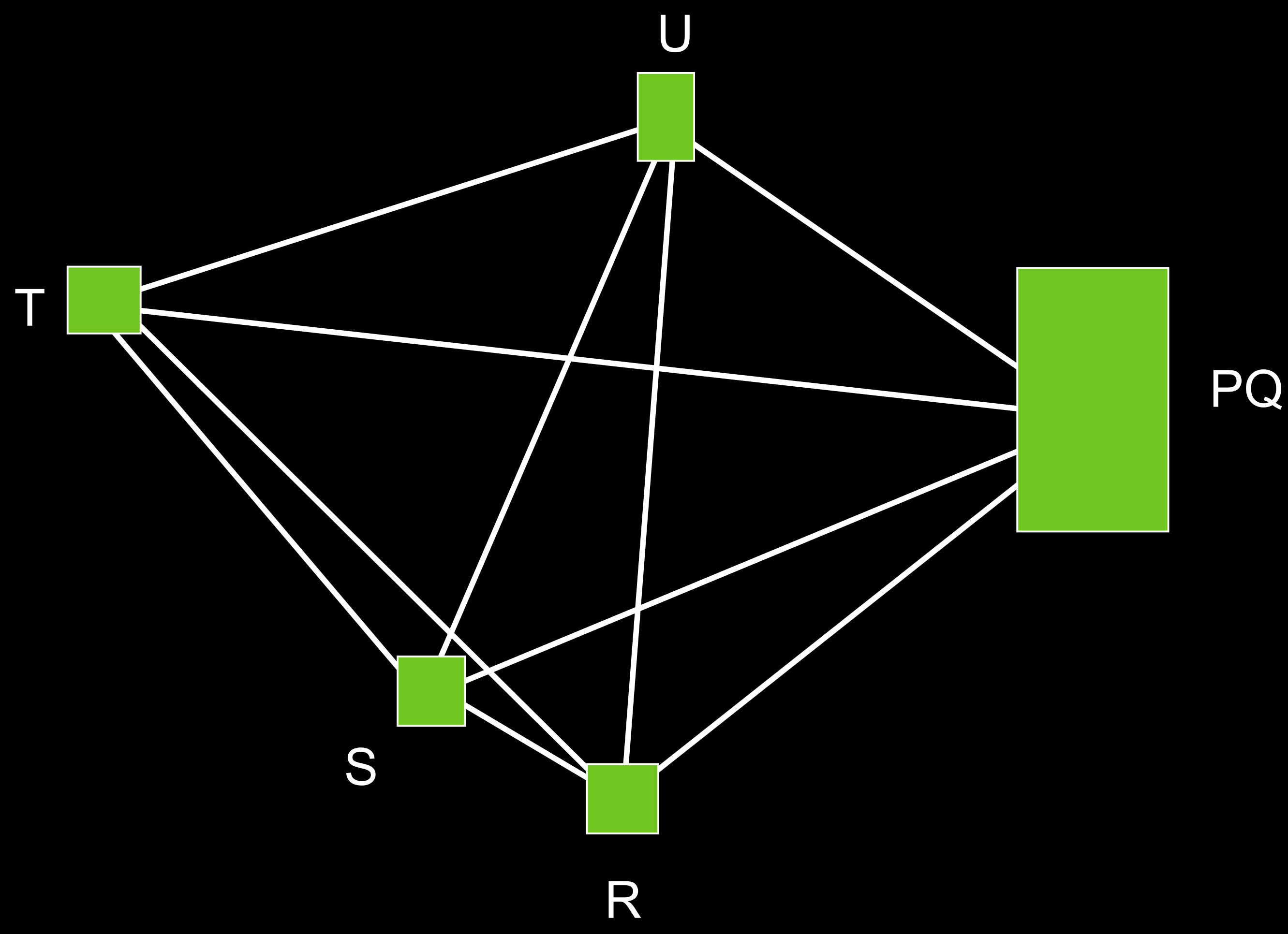

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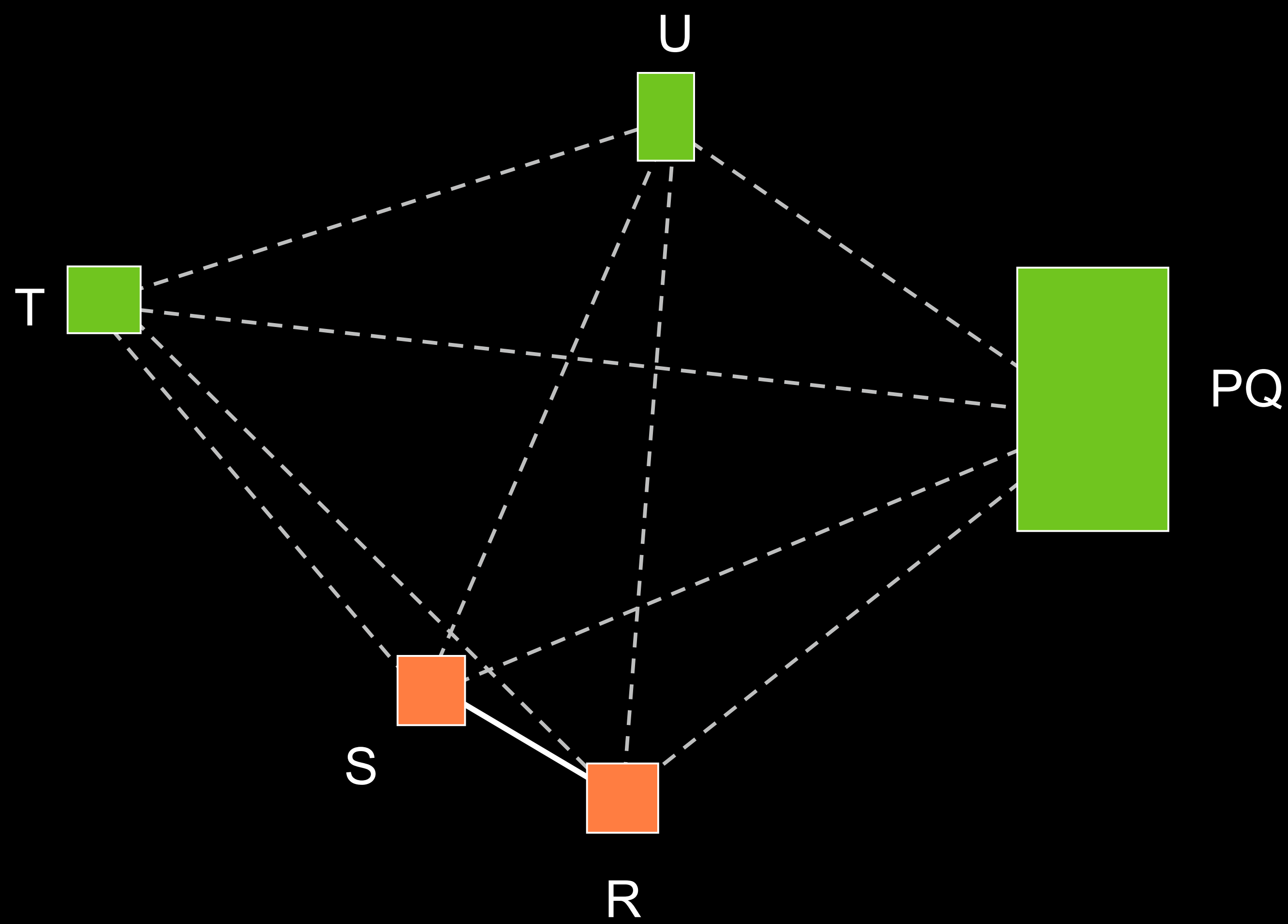

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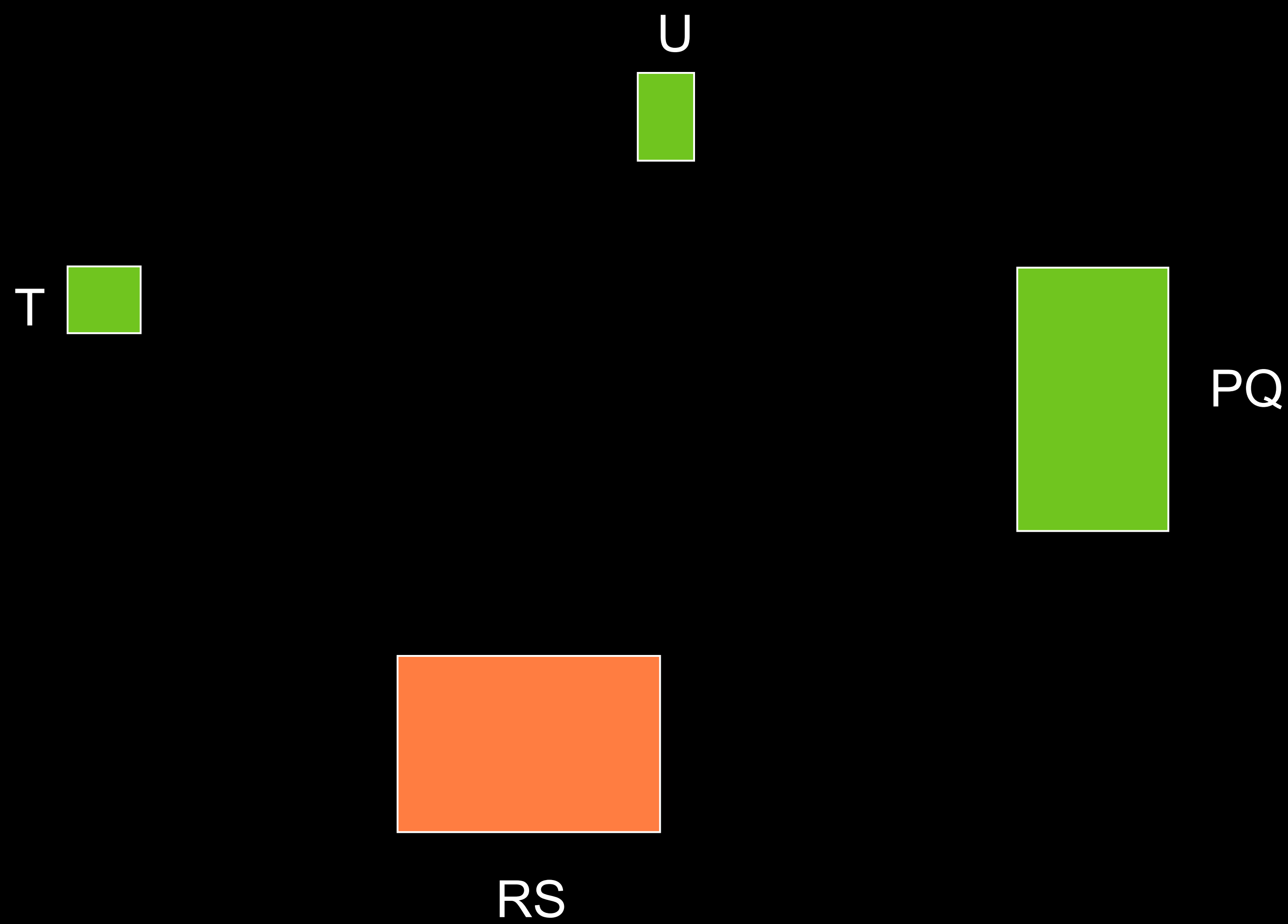


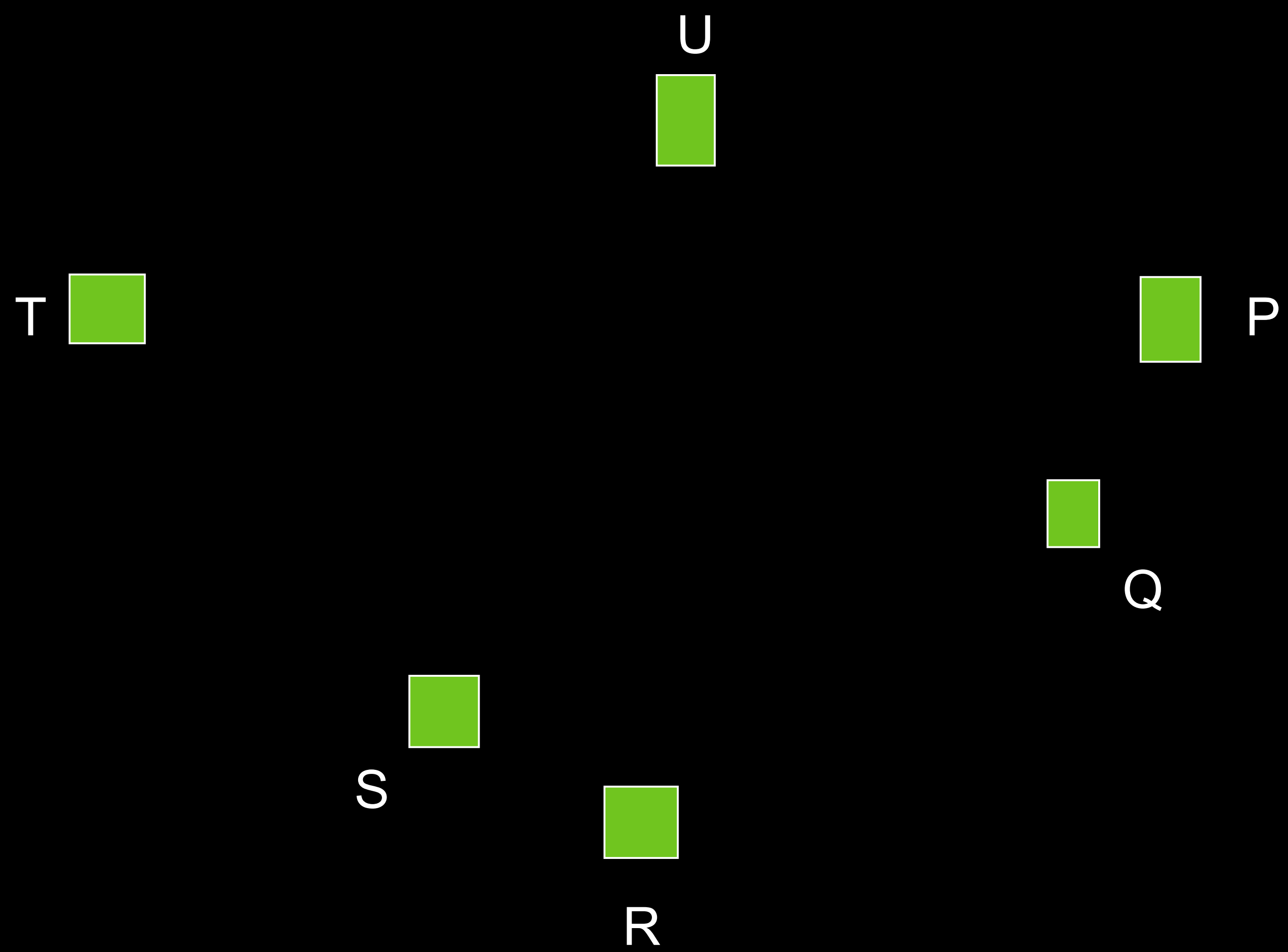
Acceleration Structures

- KD-Tree
  - Finds best match for a cluster in sub-linear time
  - Is itself a cluster tree
- Heap
  - Stores best match for each cluster
  - Enables reuse of partial results across iterations
  - Lazily updated for better performance

Heap-based Algorithm

```
Initialize KD-Tree with elements
Initialize heap with best match for each element
Repeat {
    Remove best pair <A,B> from heap
    If A and B are active clusters {
        Create new cluster C = A+B
        Update KD-Tree, removing A and B and inserting C
        Use KD-Tree to find best match for C and insert into heap
    } else if A is active cluster {
        Use KD-Tree to find best match for A and insert into heap
    }
} until only one active cluster left
```

Heap-based Algorithm Example



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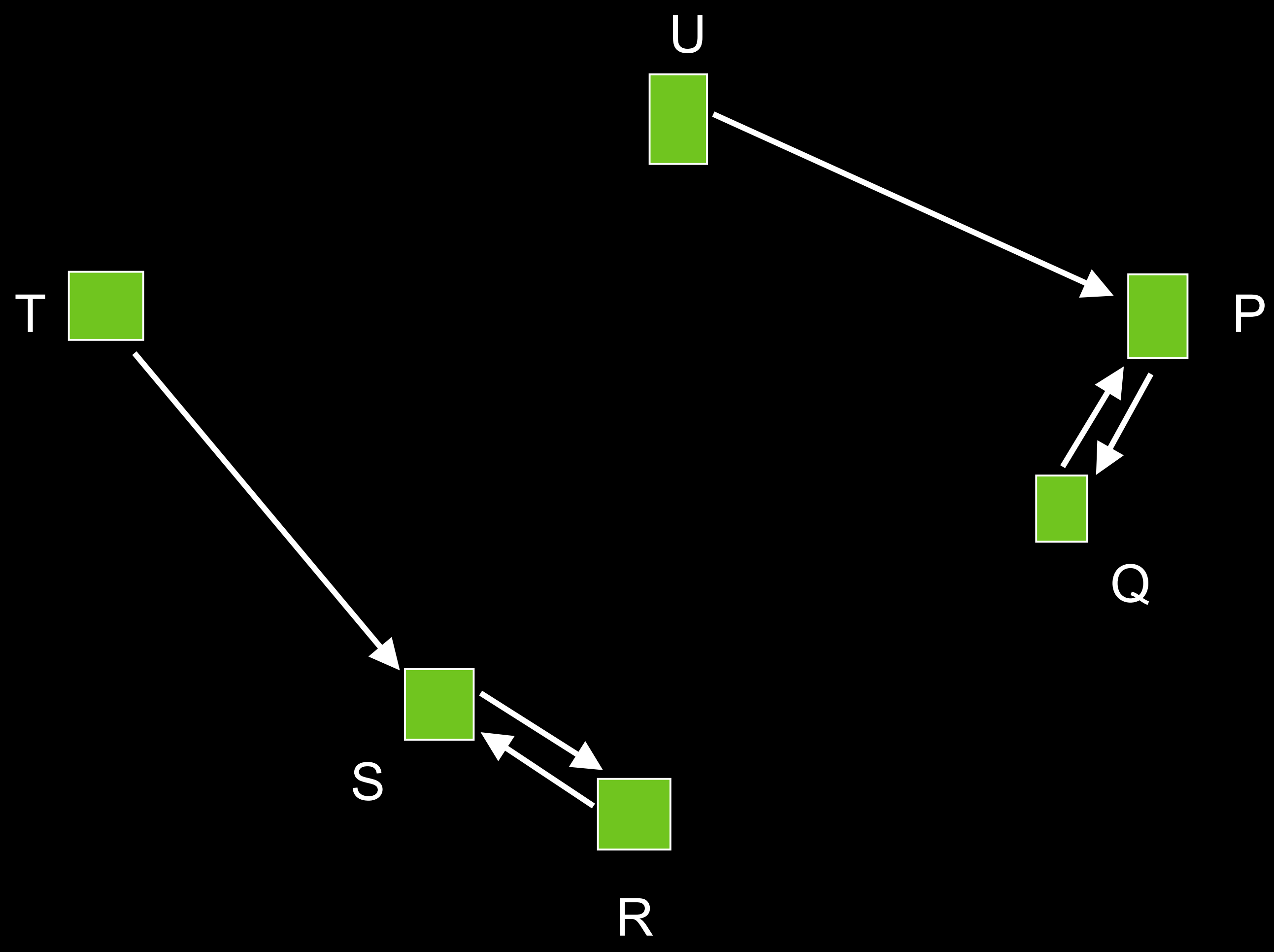

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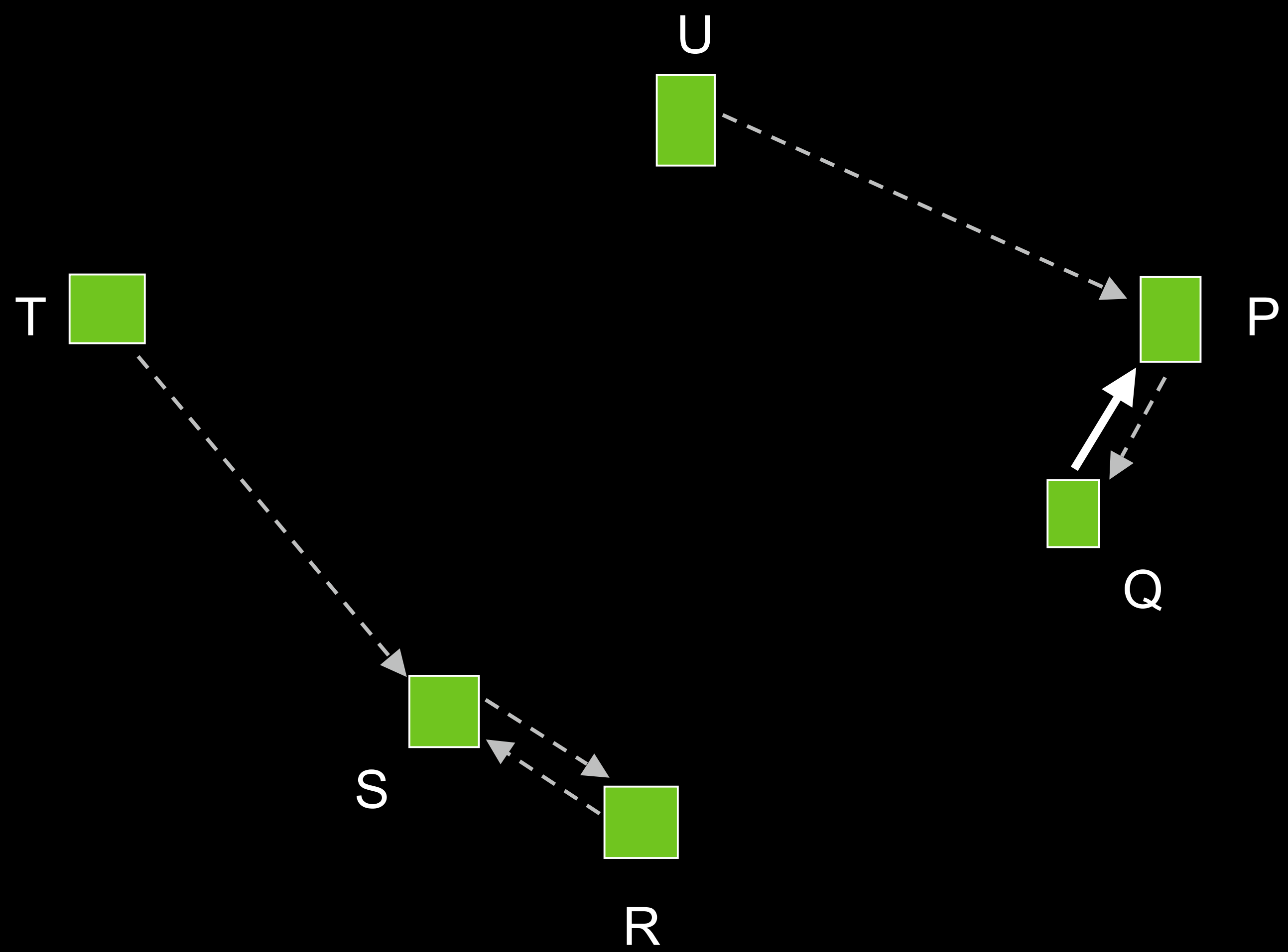

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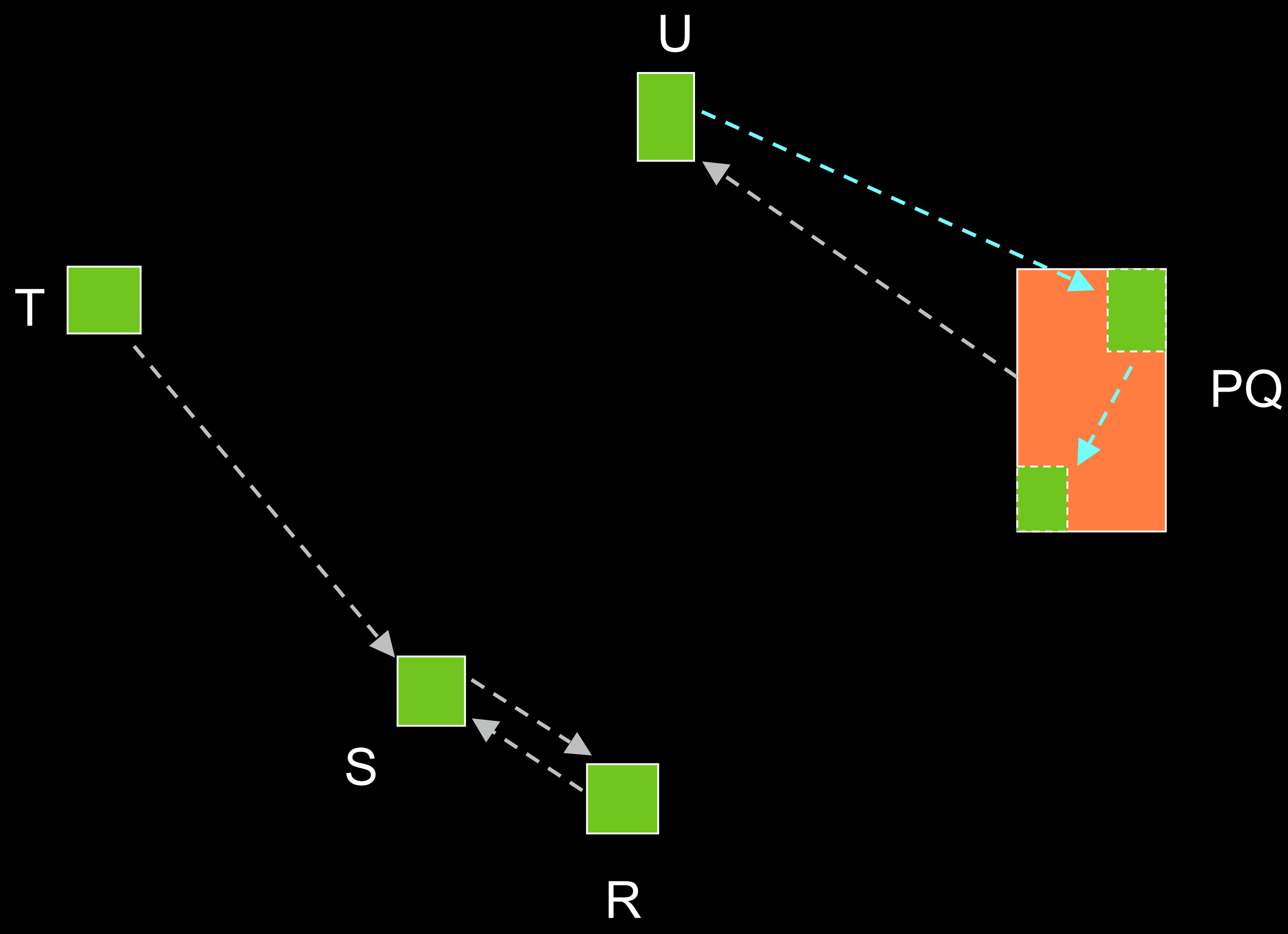

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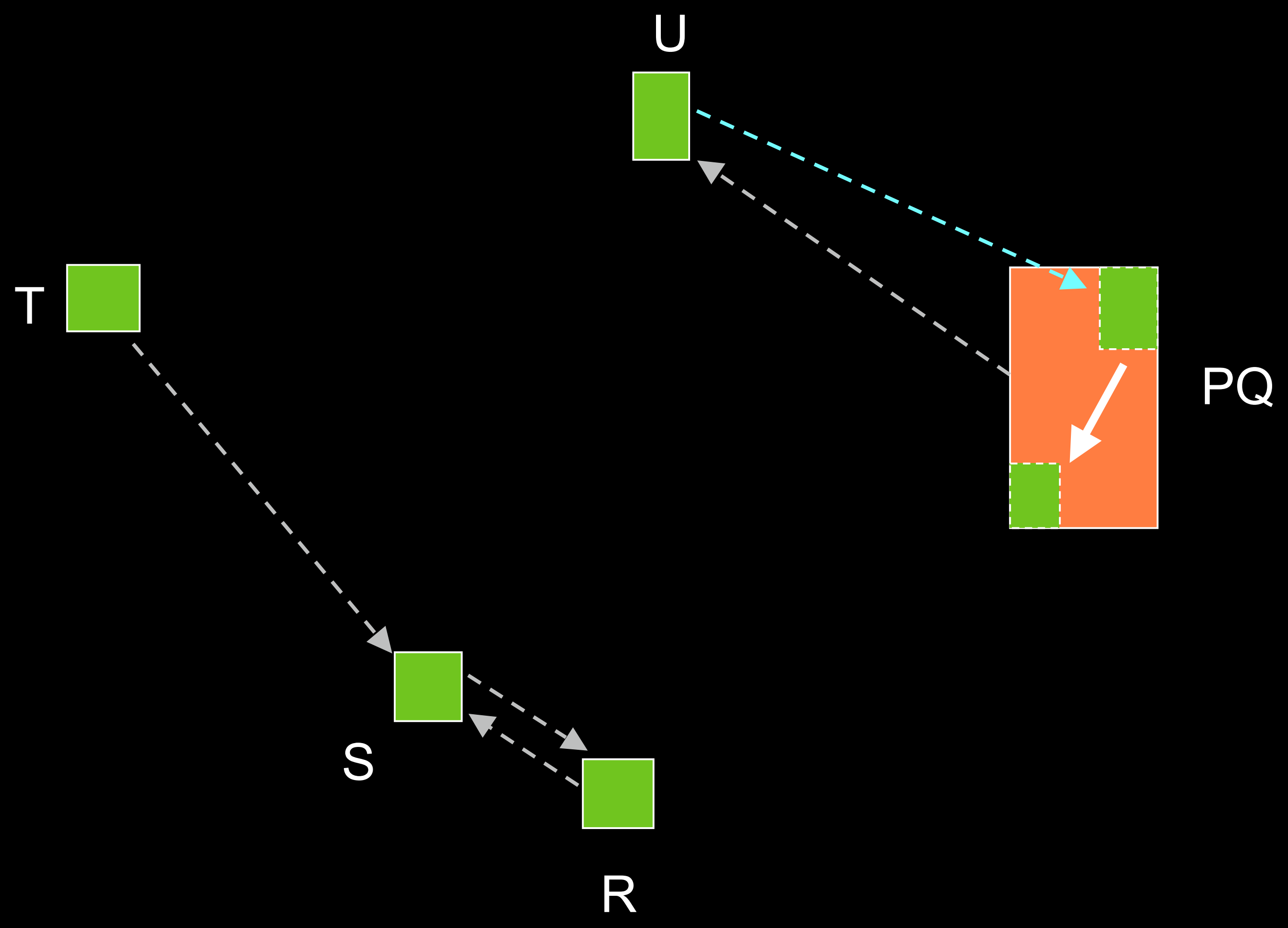

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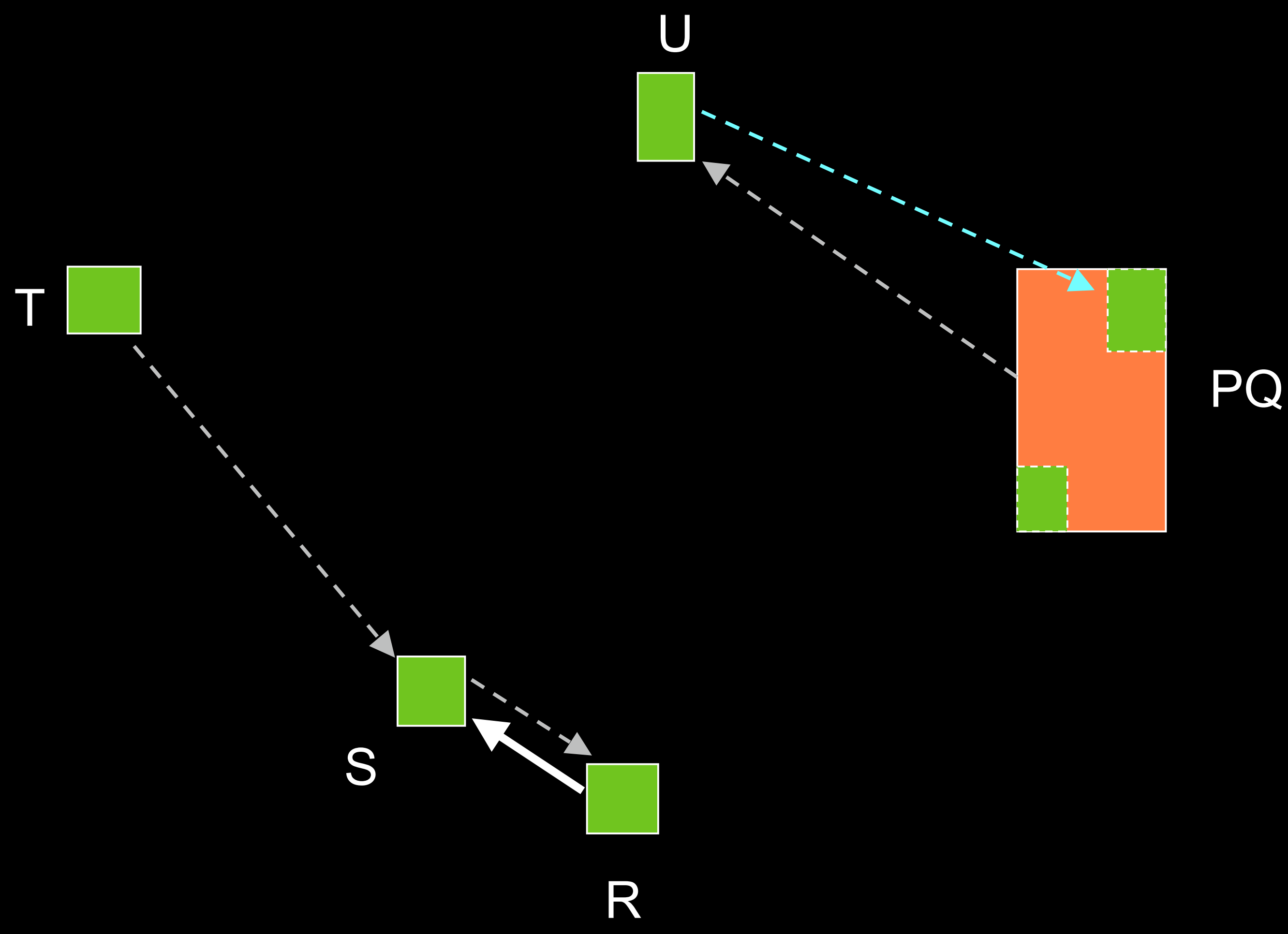

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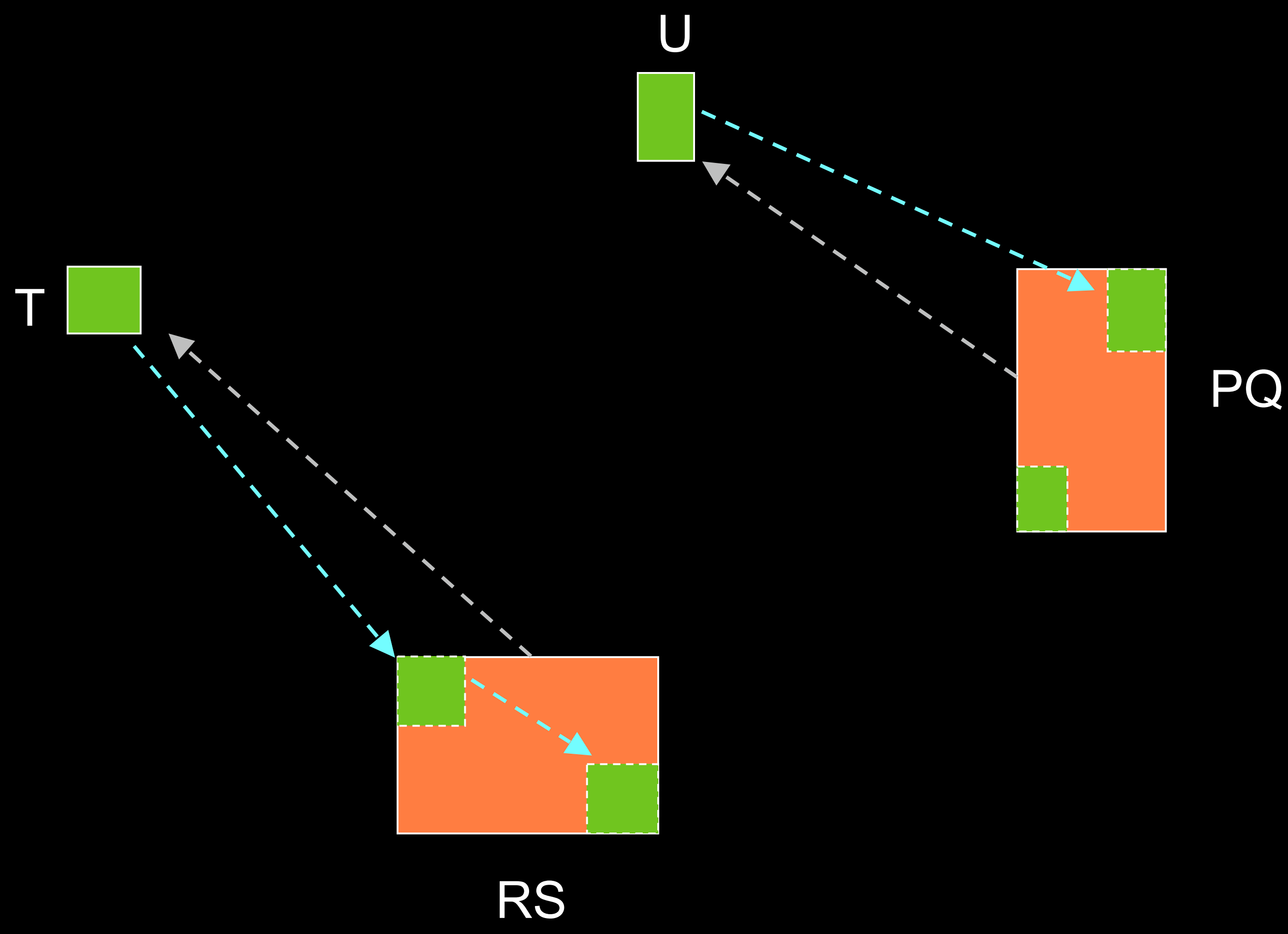
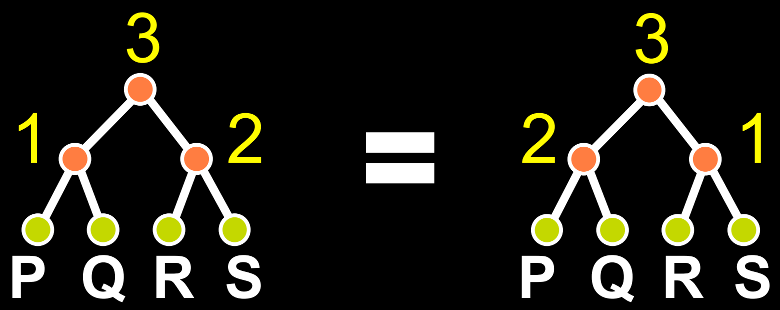


Locally-ordered Insight

- Can build the exactly same tree in different order

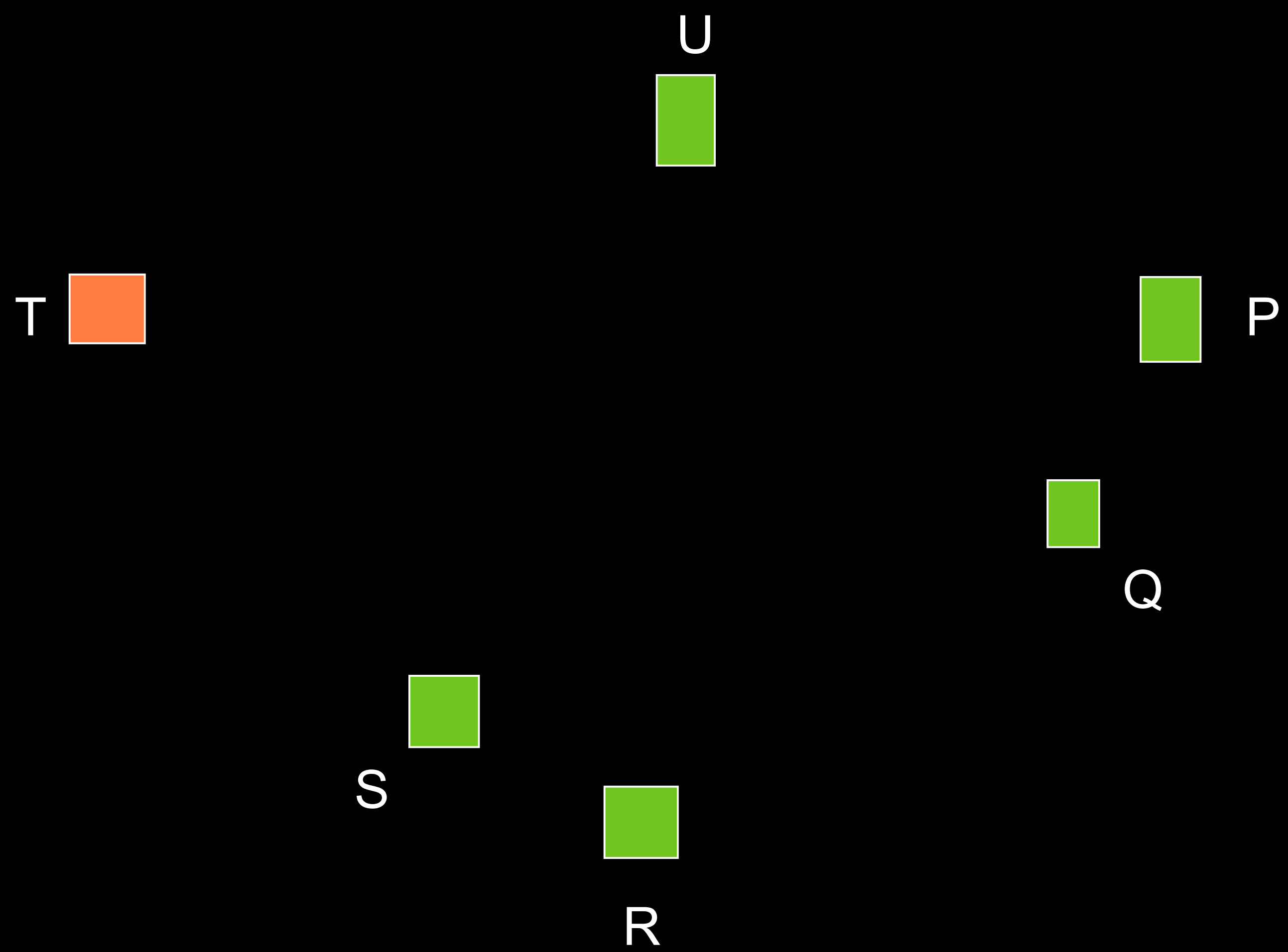


- How can we use this insight?
  - If d(A,B) is non-decreasing, meaning d(A,B) <= d(A,B+C)
  - And A and B are each others best match
  - Greedy algorithm must cluster A and B eventually
  - So cluster them together immediately

Locally-ordered Algorithm

```
Initialize KD-Tree with elements
Select an element A and find its best match B using KD-Tree
Repeat {
    Let C = best match for B using KD-Tree
    If d(A,B) == d(B,C) {    //usually means A==C
        Create new cluster D = A+B
        Update KD-Tree, removing A and B and inserting D
        Let A = D and B = best match for D using KD-Tree
    } else {
        Let A = B and B = C
    }
} until only one active cluster left
```

Locally-ordered Algorithm Example



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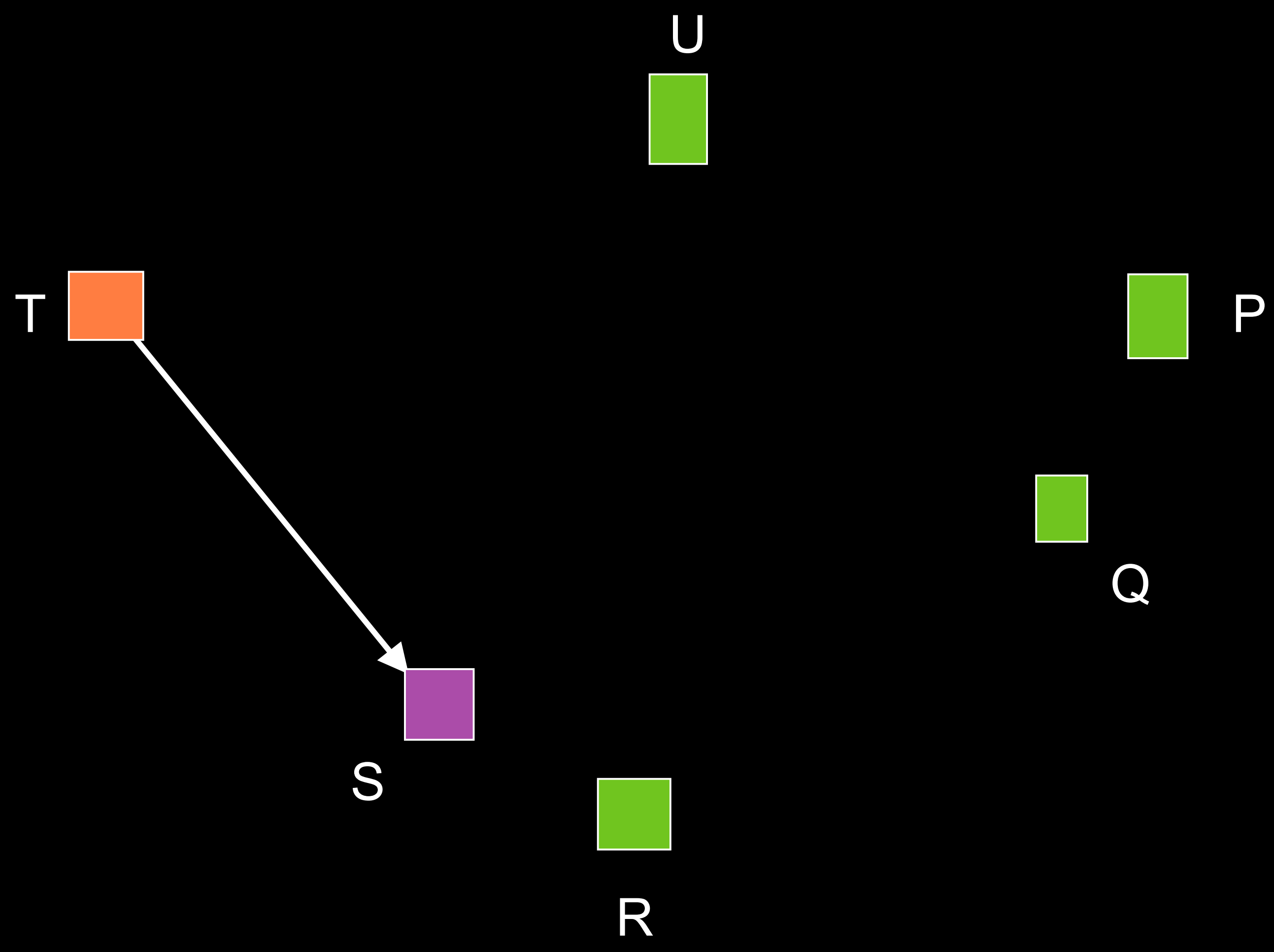

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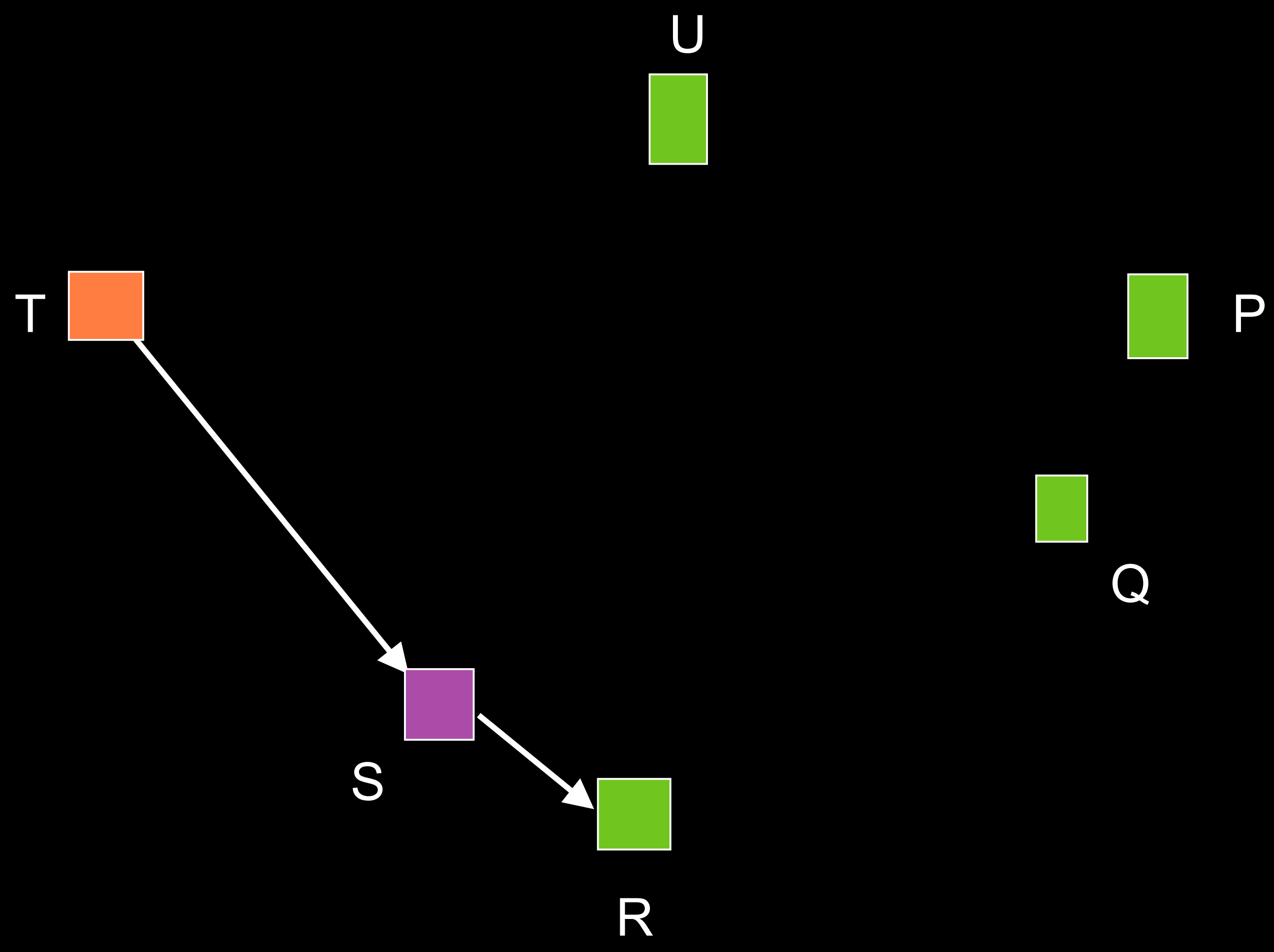

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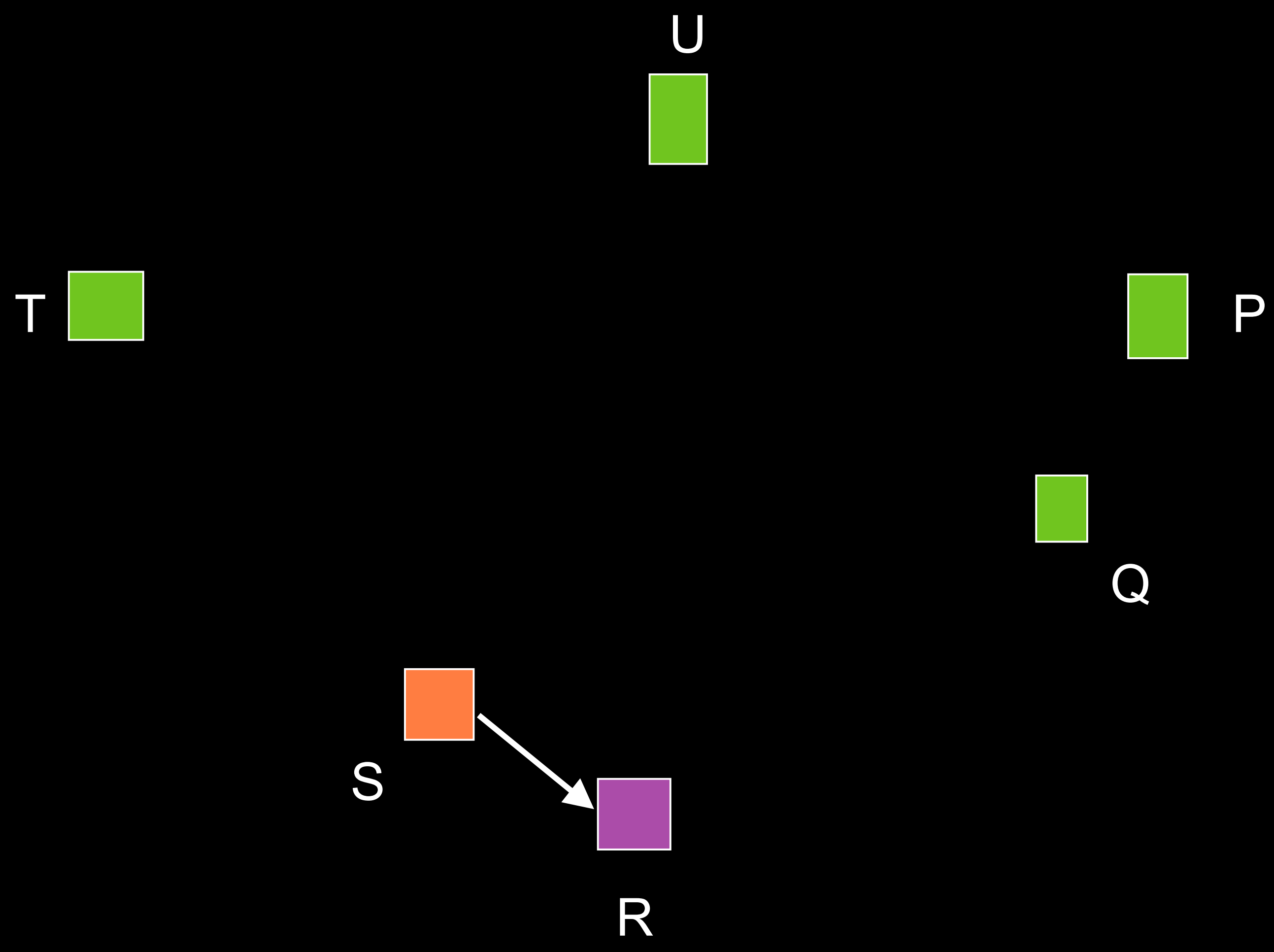

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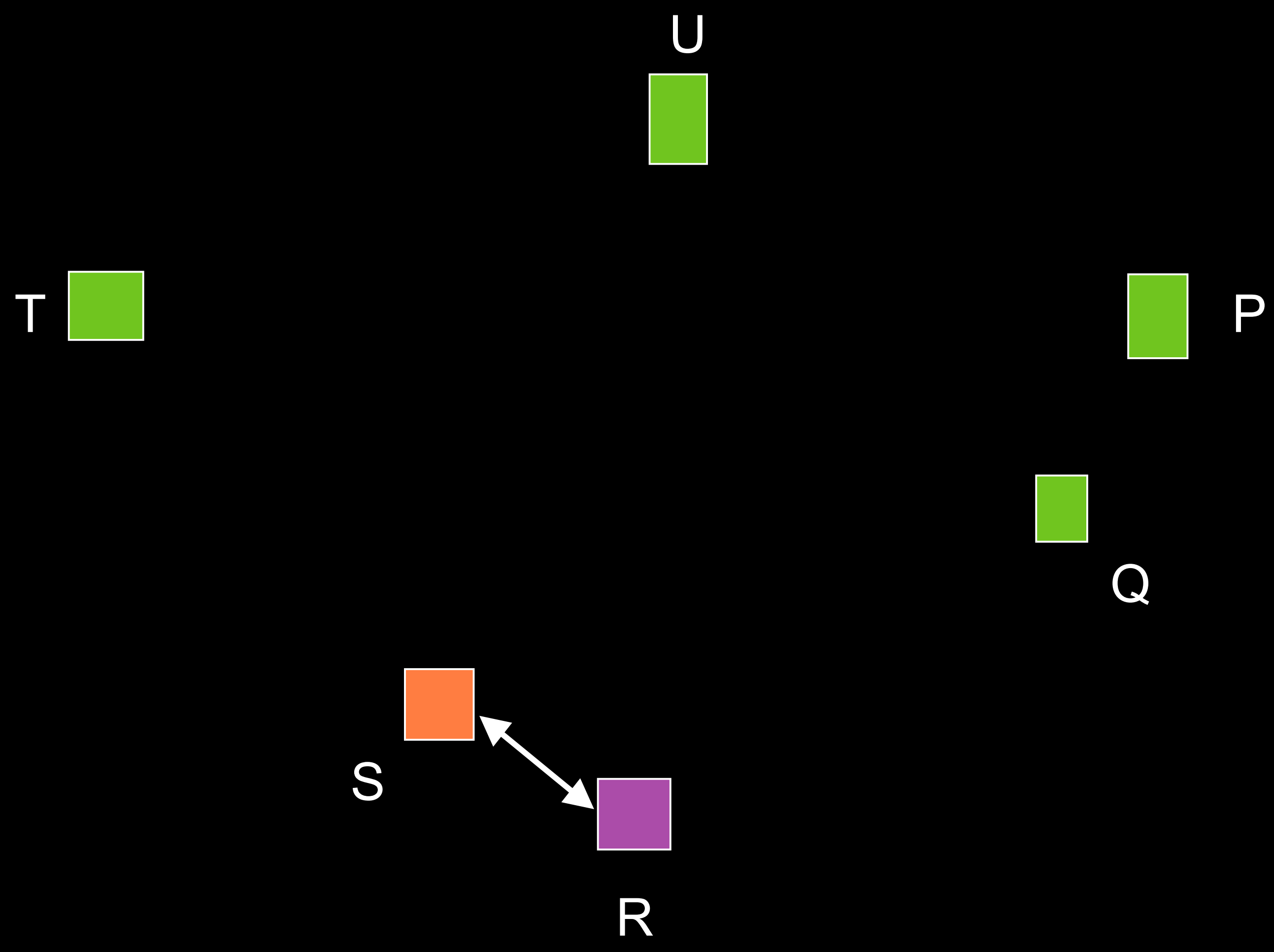

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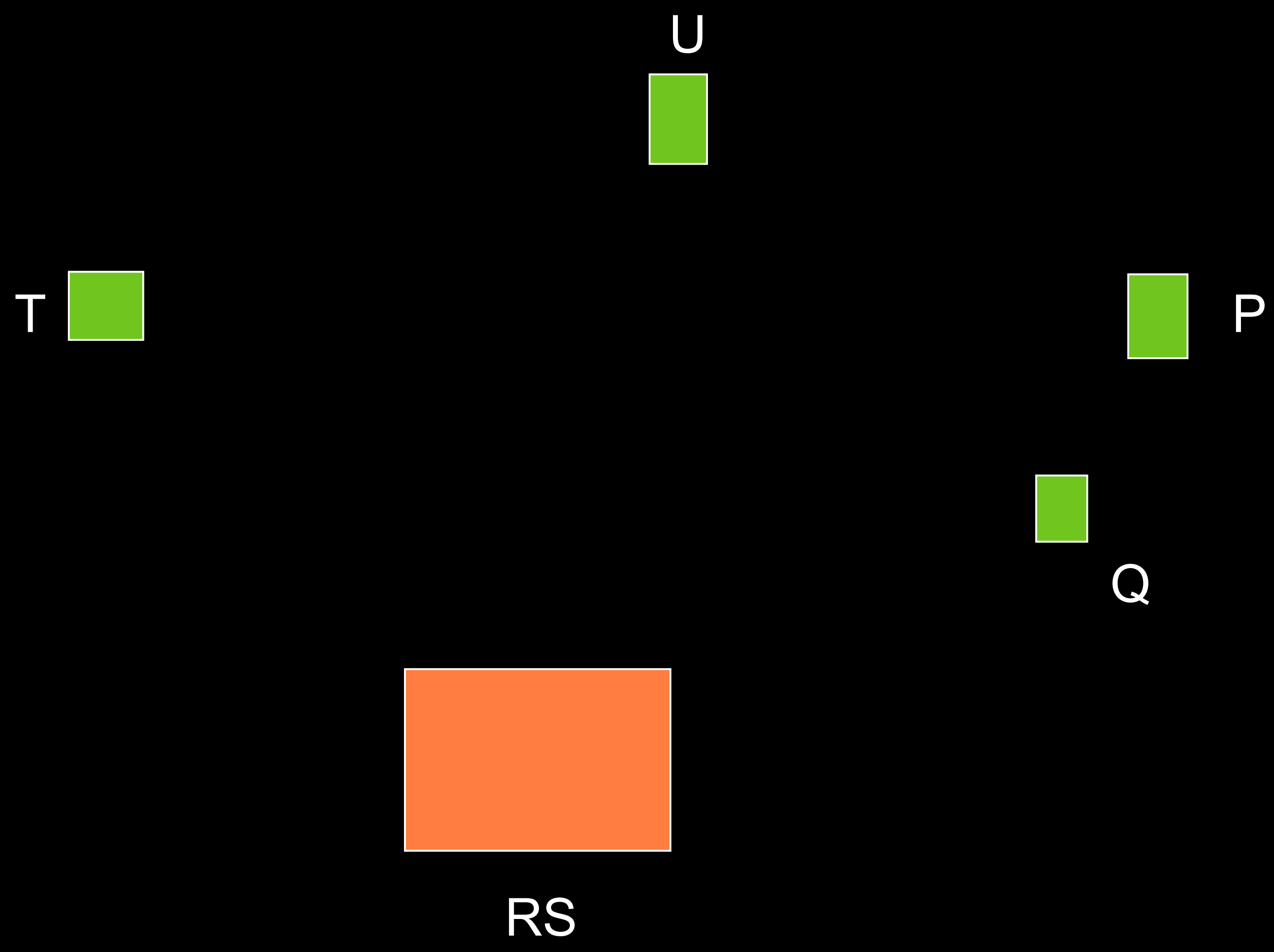

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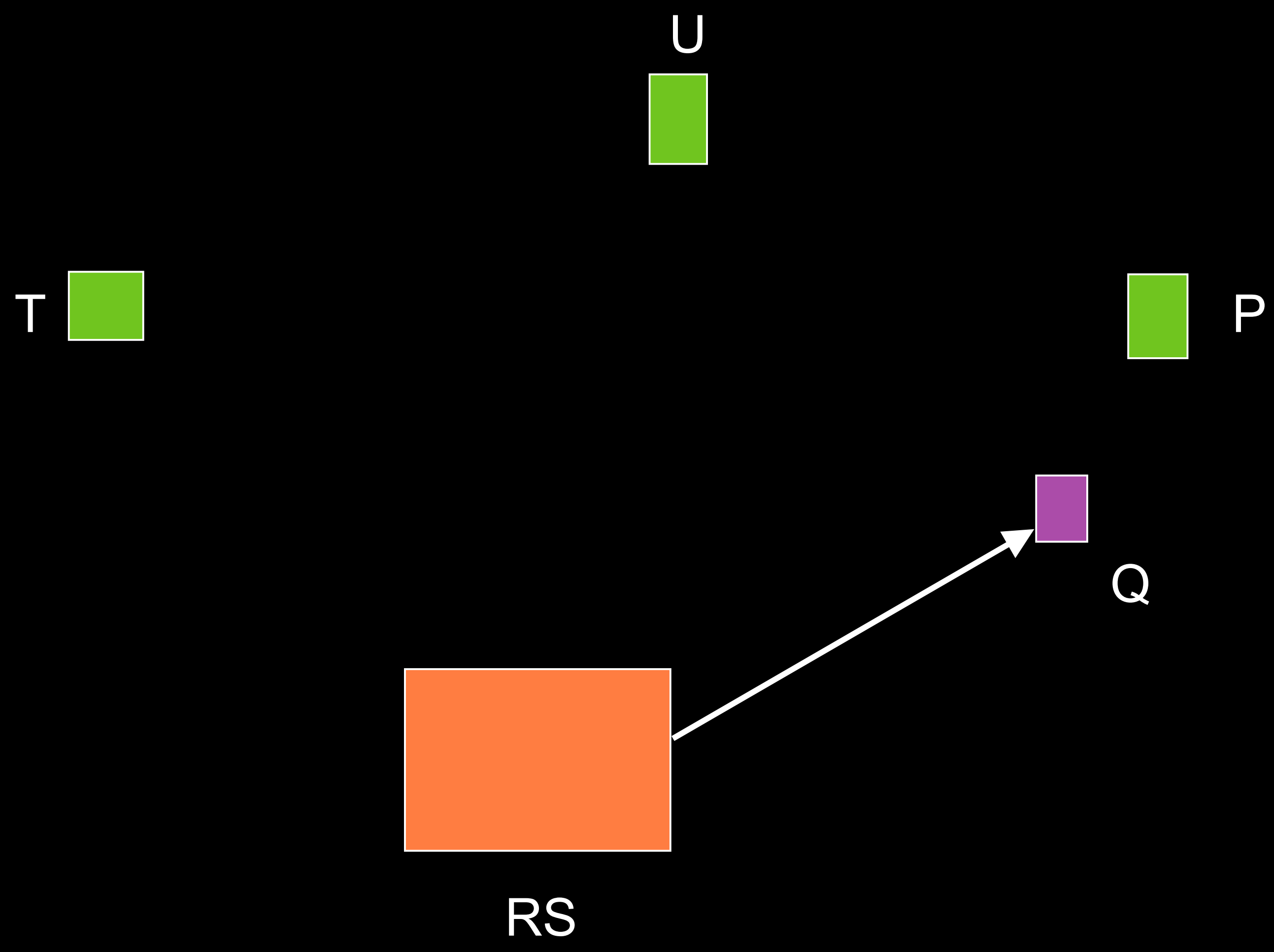

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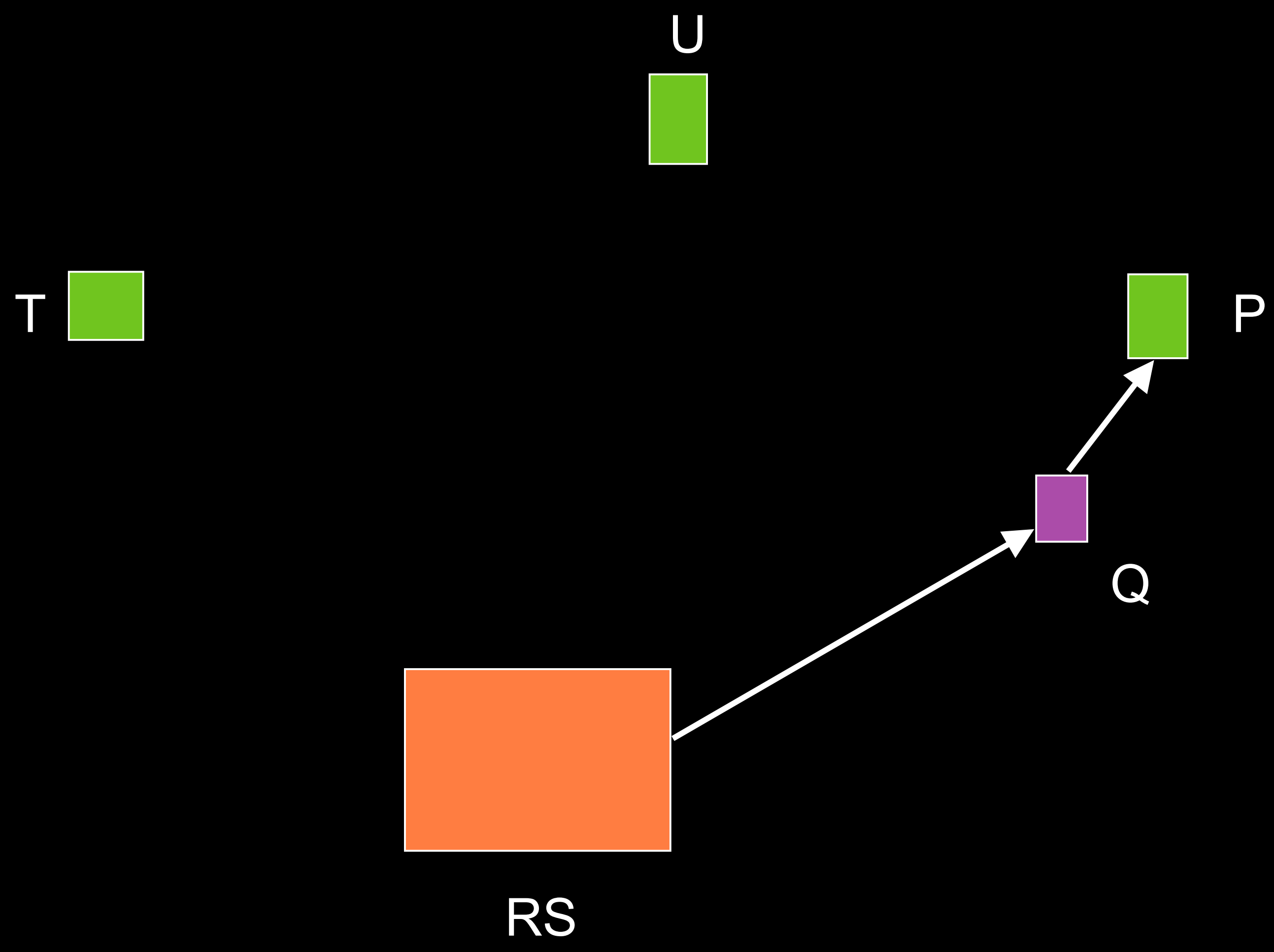

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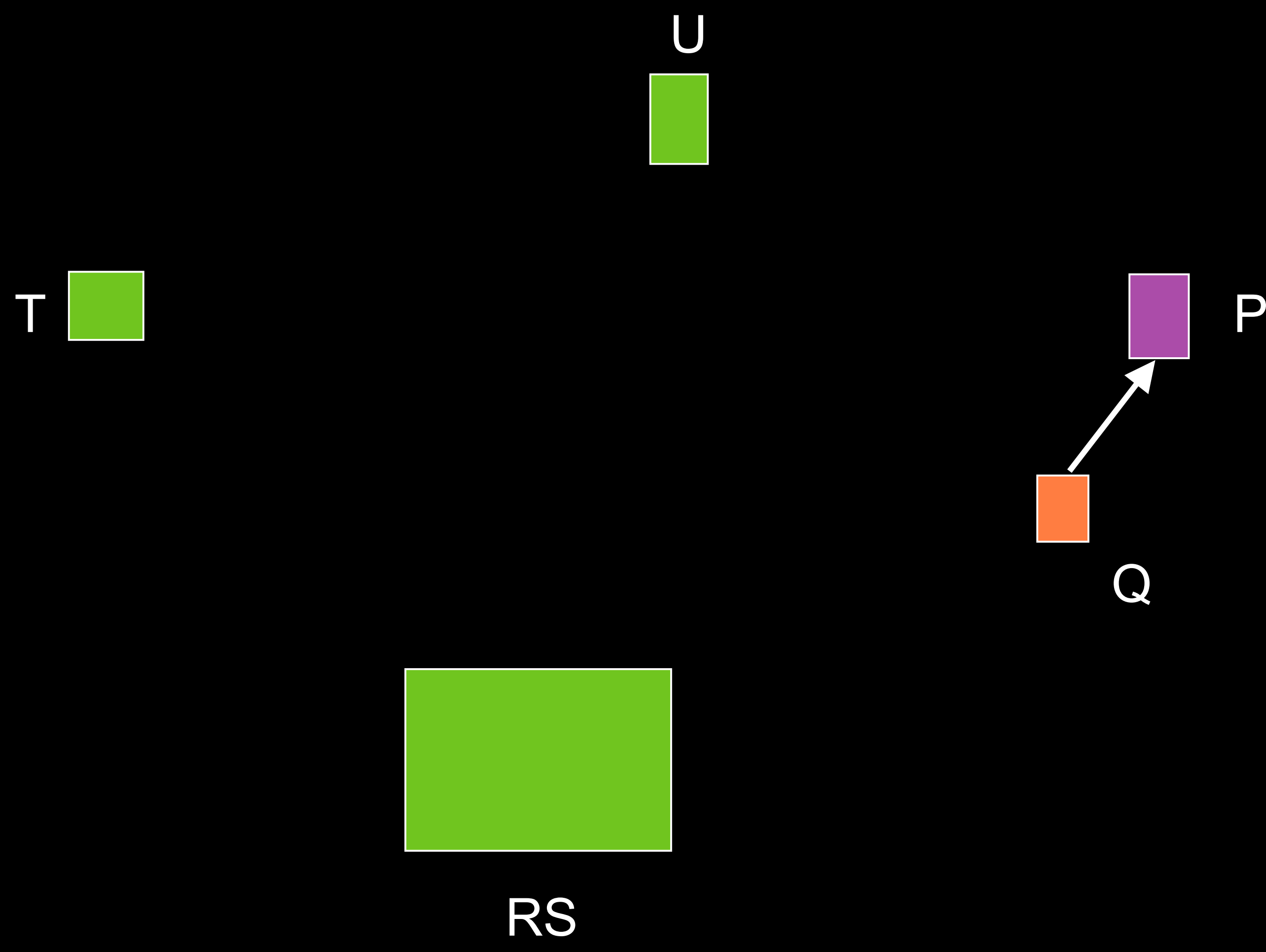

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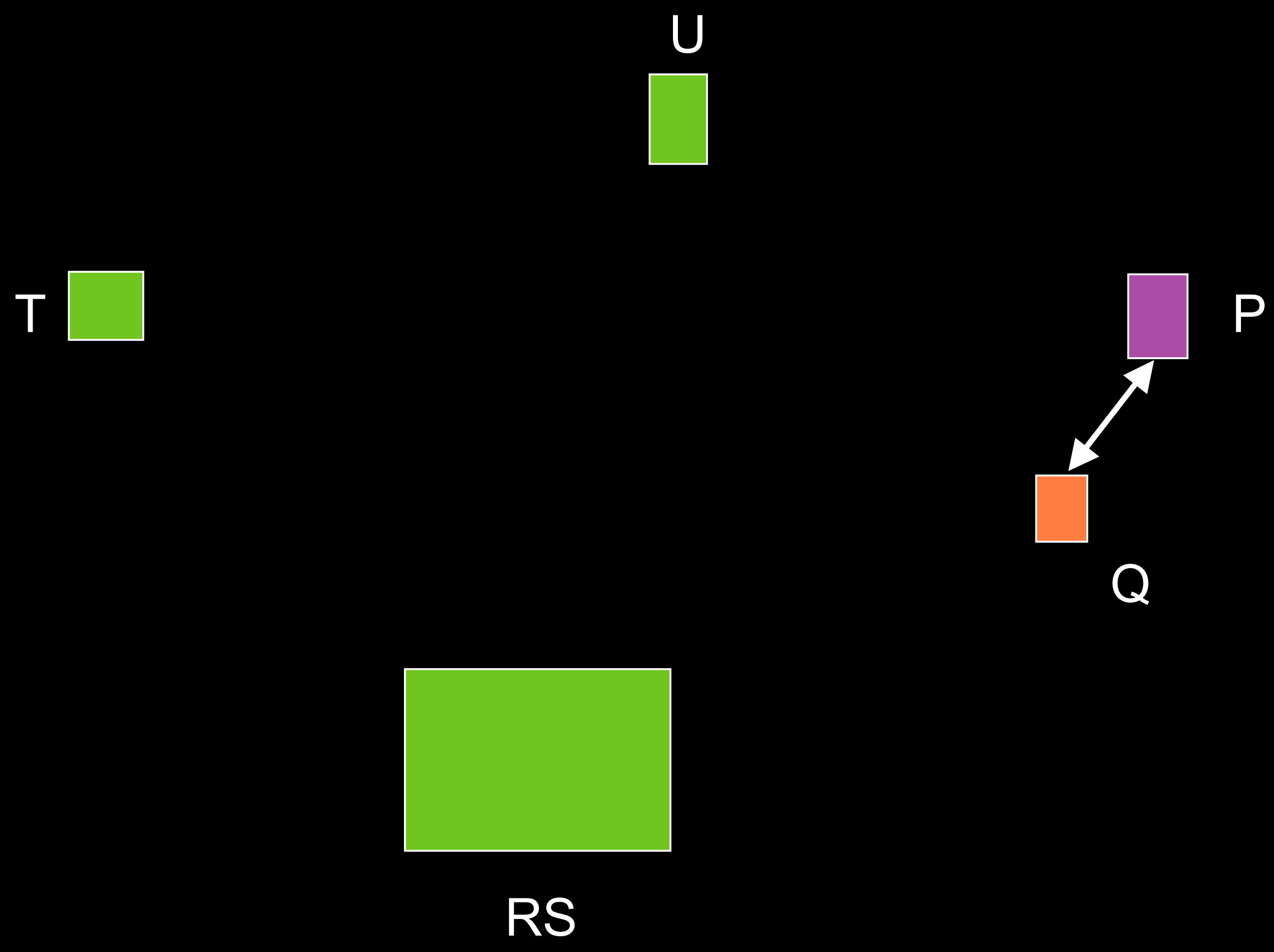

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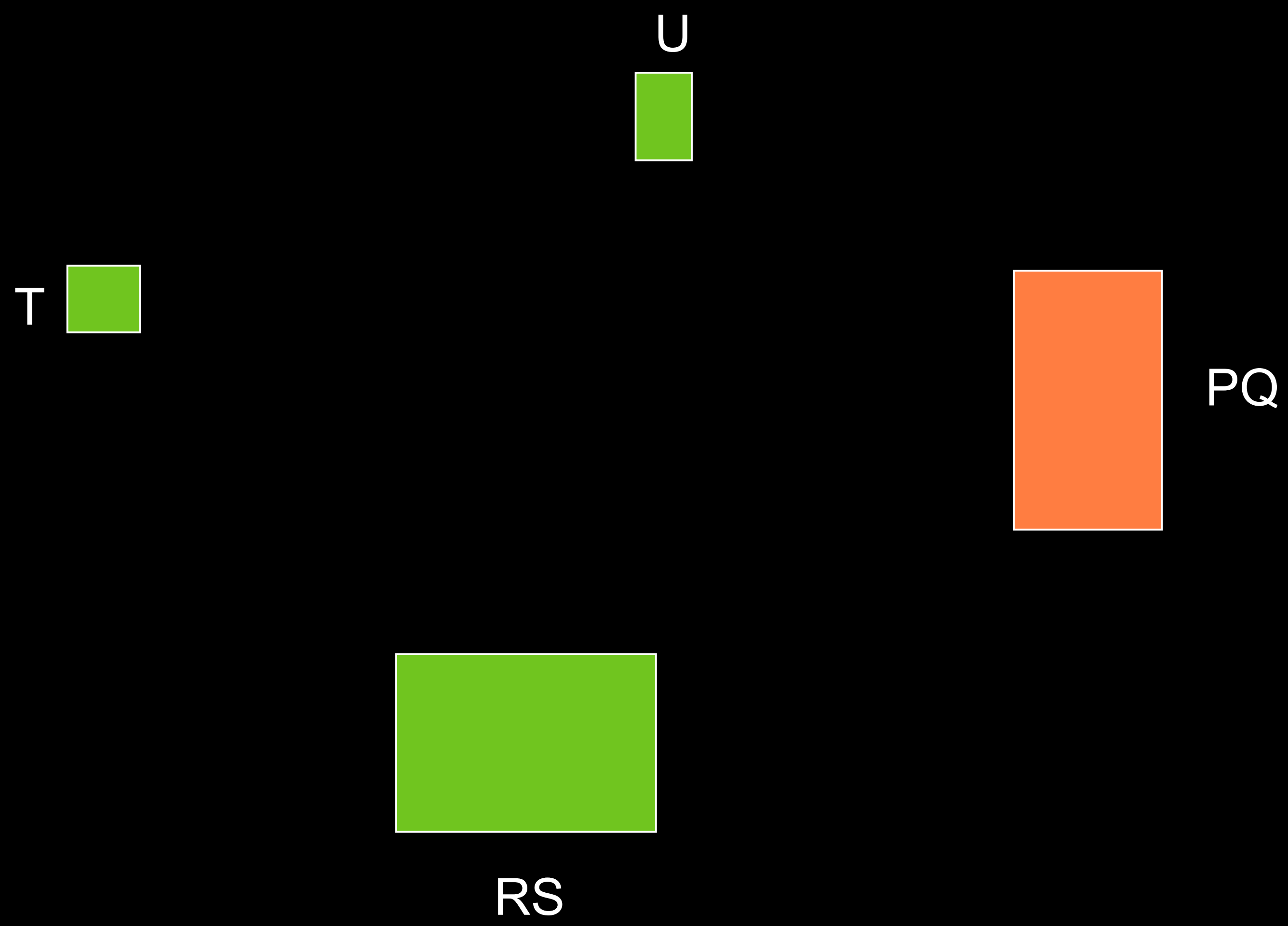


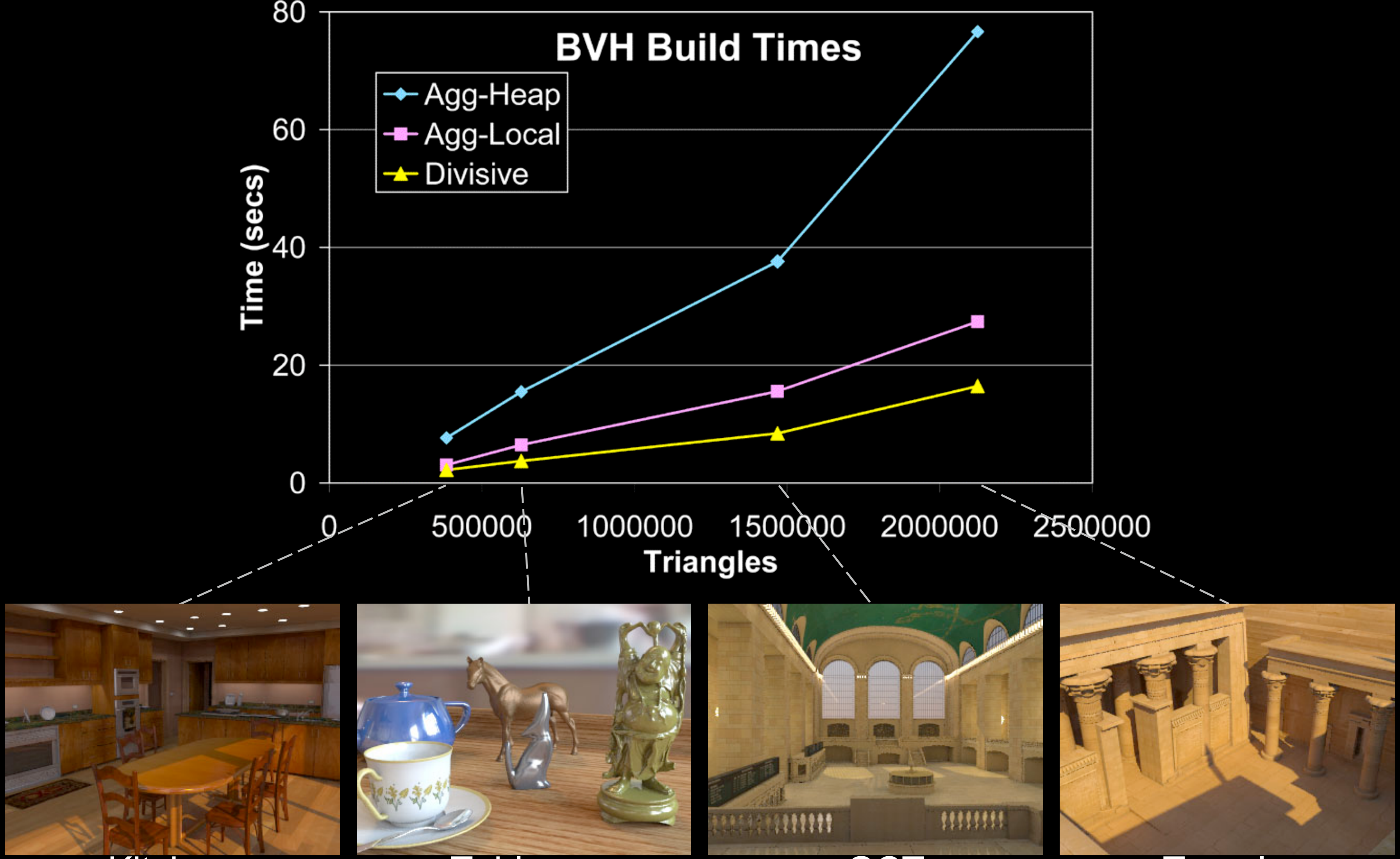
Locally-ordered Algorithm

- Roughly 2x faster than heap-based algorithm
  - Eliminates heap
  - Better memory locality
  - Easier to parallelize
  - But d(A,B) must be non-decreasing

Results: BVH

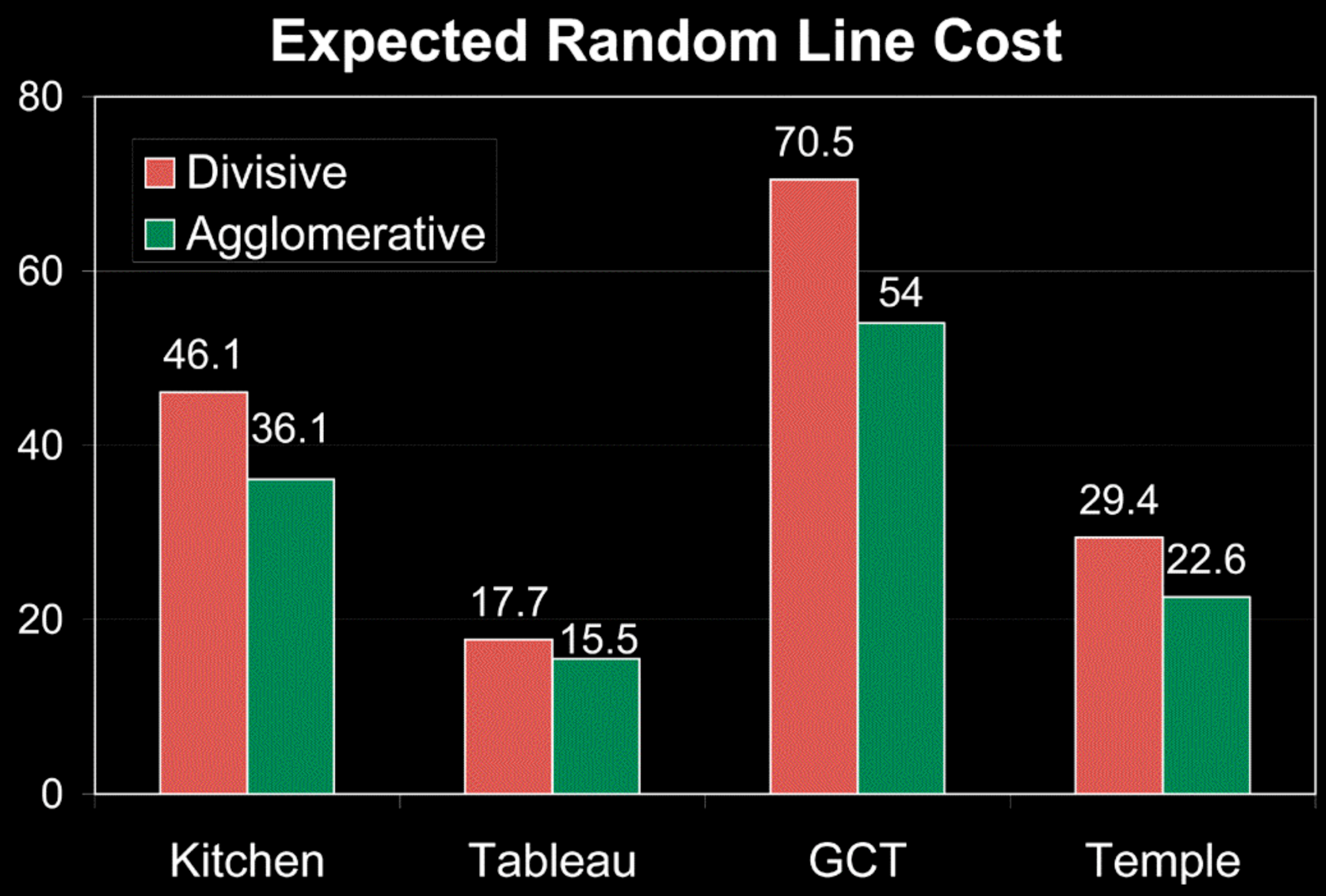
- BVH – Binary tree of axis-aligned bounding boxes
- Divisive [from Wald 07]
  - Evaluate 16 candidate splits along longest axis per step
  - Surface area heuristic used to select best one
- Agglomerative
  - d(A,B) = surface area of bounding box of A+B
- Used Java 1.6JVM on 3GHz Core2 with 4 cores
  - No SIMD optimizations, packets tracing, etc.

Results: BVH



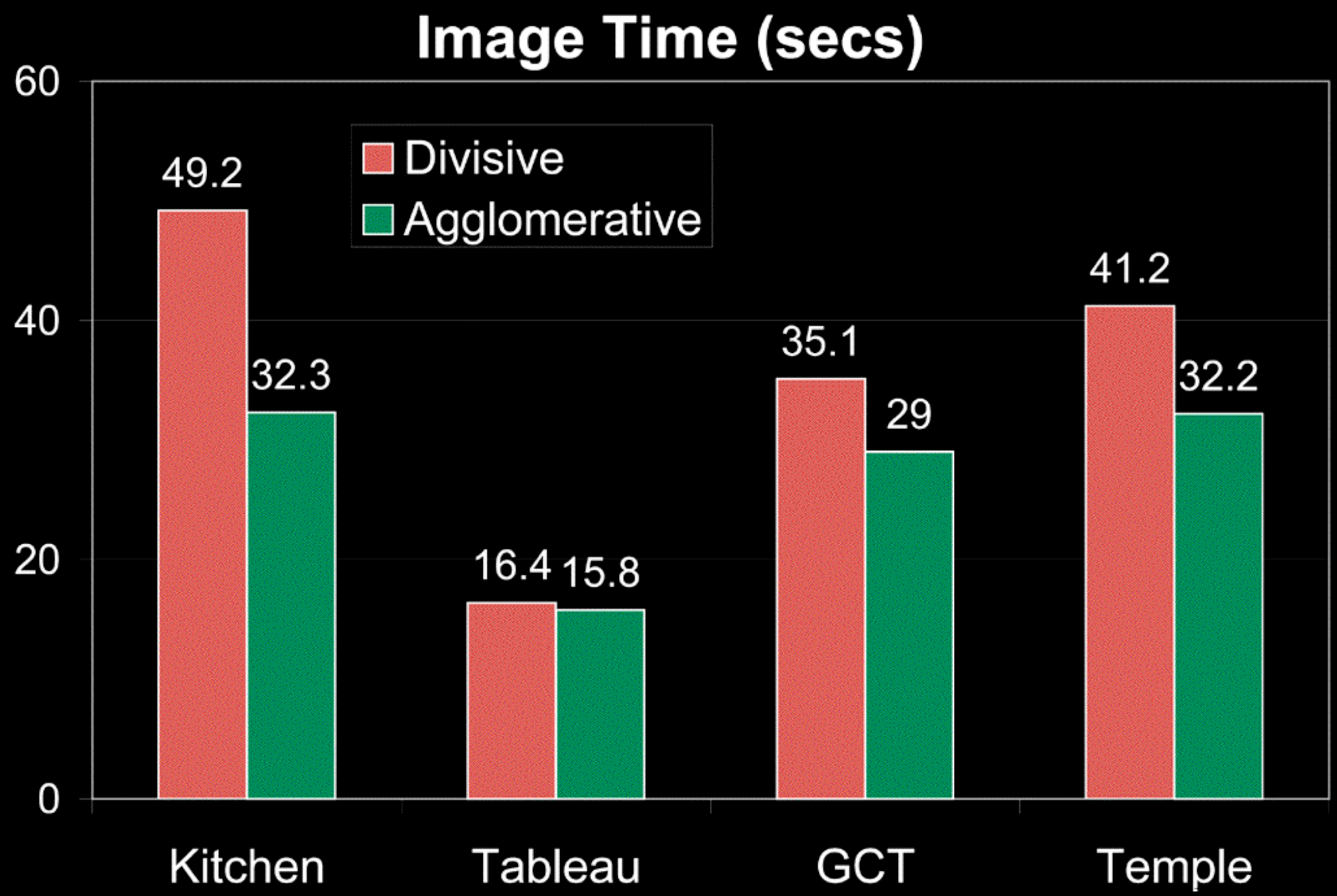
Kitchen Tableau GCT Temple

Results: BVH



Surface area heuristic with triangle cost = 1 and box cost = 0.5

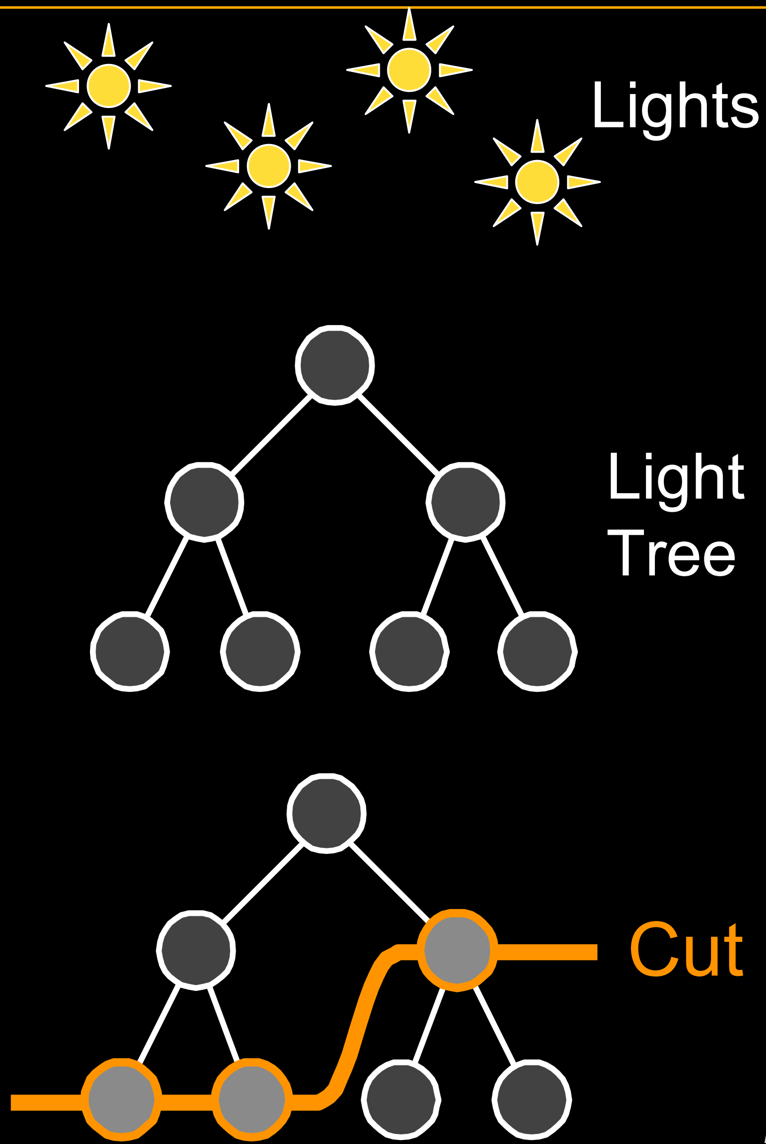
Results: BVH



1280x960 Image with 16 eye and 16 shadow rays per pixel, without build time

Lightcuts Key Concepts

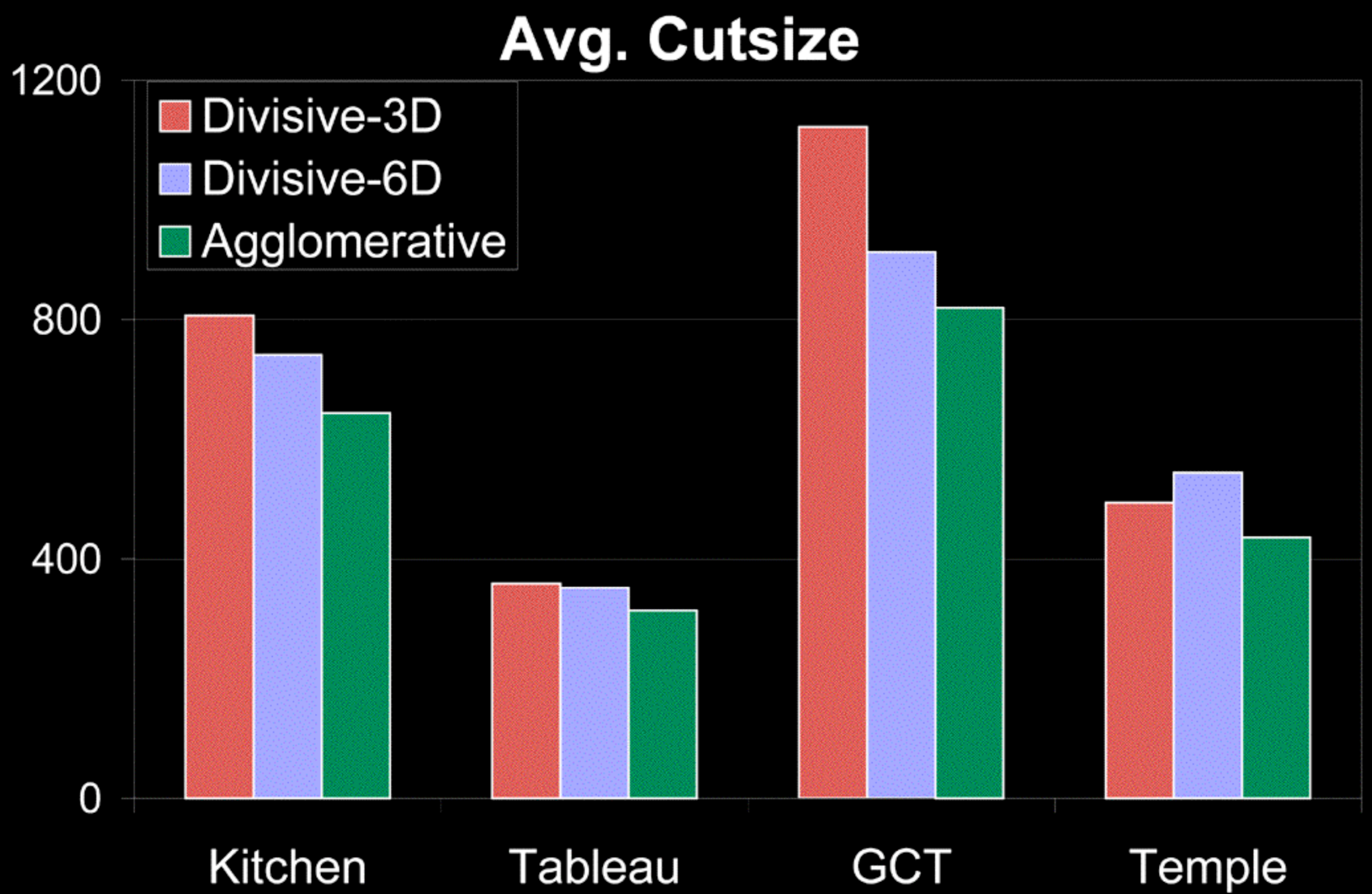
- Unified representation
  - Convert all lights to points
    - ~200,000 in examples
- Build light tree
  - Originally agglomerative
- Adaptive cut
  - Partitions lights into clusters
  - Cutsize = # nodes on cut



Lightcuts

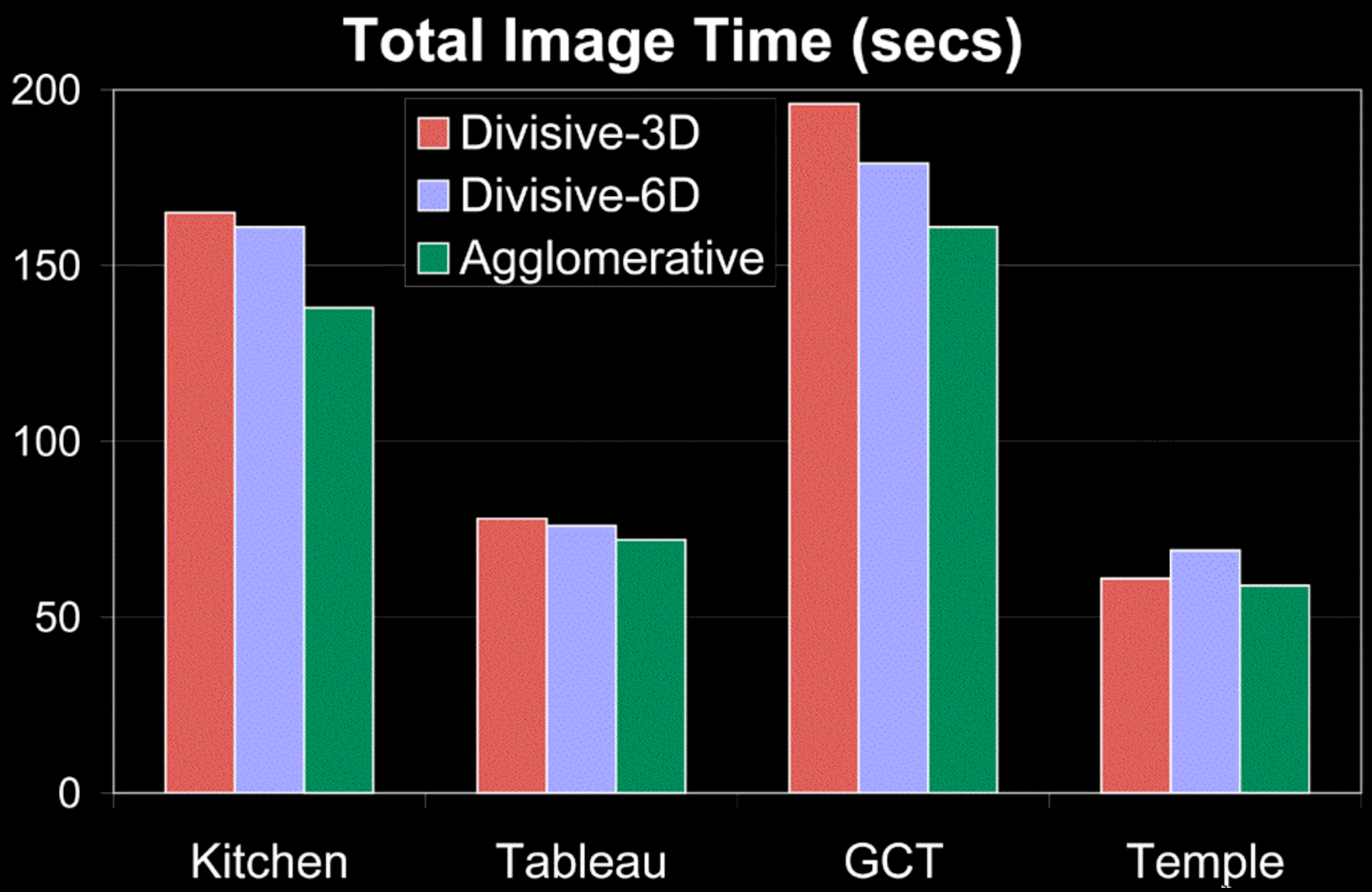
- Divisive
  - Split middle of largest axis
  - Two versions
    - 3D – considers spatial position only
    - 6D – considers position and direction
- Agglomerative
  - New dissimilarity function, d(A,B)
    - Considers position, direction, and intensity

Results: Lightcuts



640x480 image with 16x antialiasing and ~200,000 point lights

Results: Lightcuts



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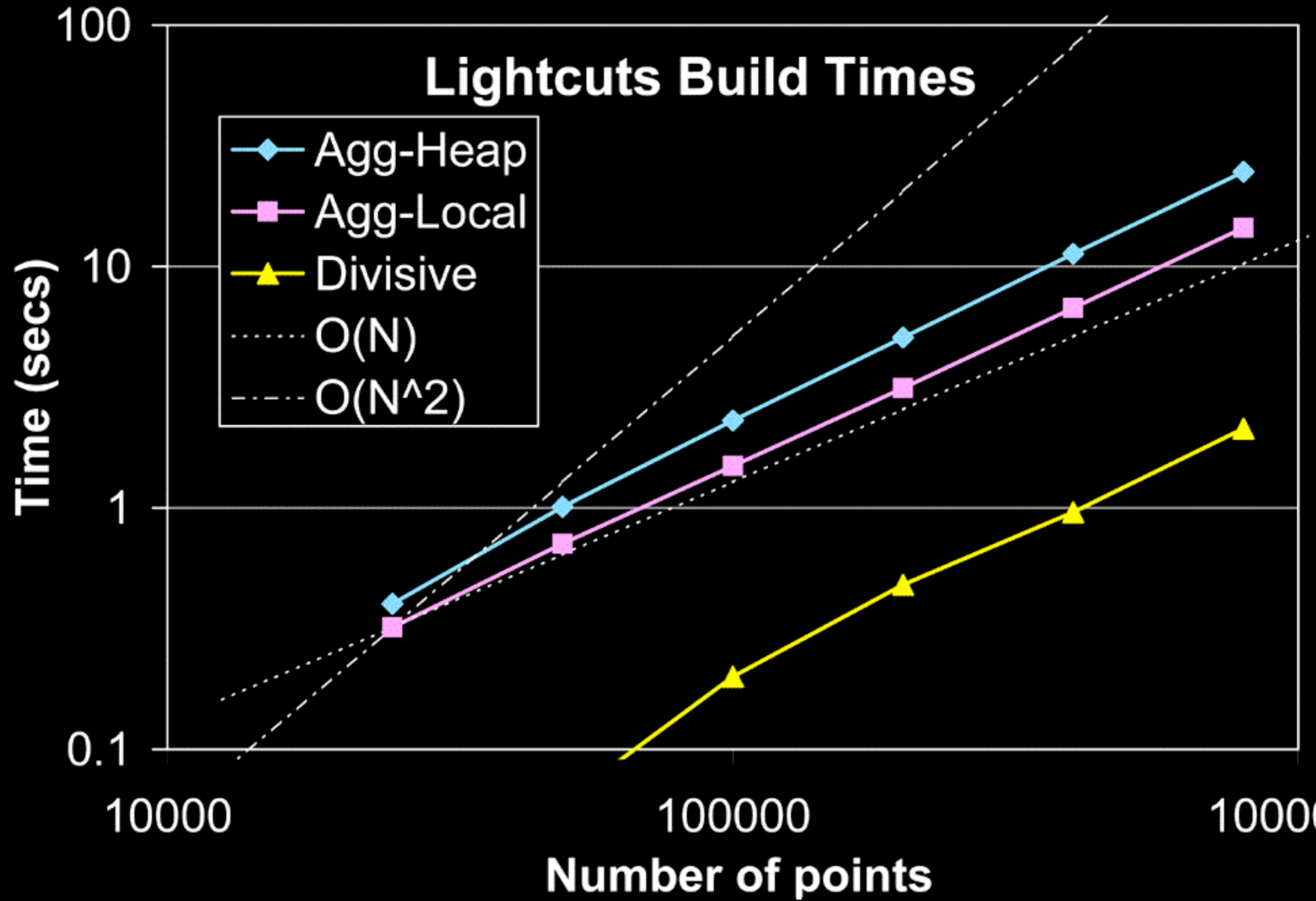

Kitchen model with varying numbers of indirect lights

Conclusions

- Agglomerative clustering is a viable alternative
  - Two novel fast construction algorithms
    - Heap-based algorithm
    - Locally-ordered algorithm
  - Tree quality is often superior to divisive
  - Dissimilarity function d(A,B) is very flexible

- Future work
  - Find more applications that can leverage this flexibility

Acknowledgements

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  - Jeremiah Fairbanks, Moreno Piccolotto, Veronica Sundstedt & Bristol Graphics Group,
- Support
  - NSF, IBM, Intel, Microsoft